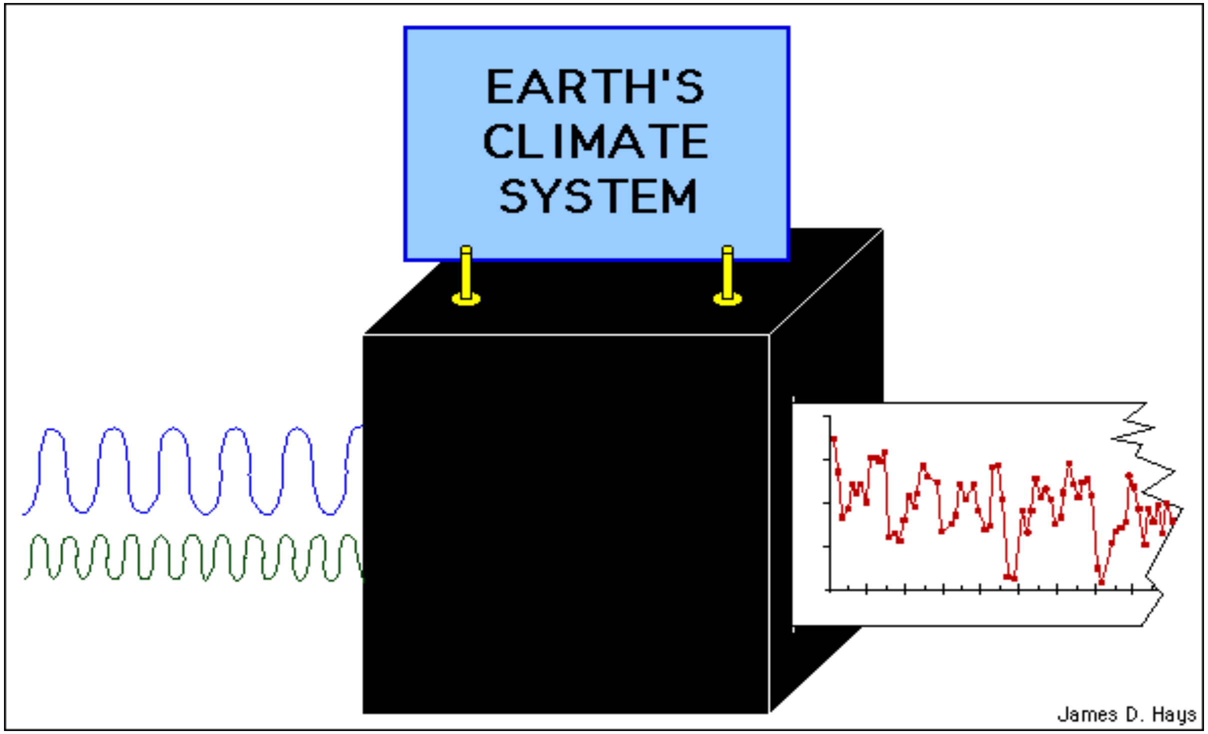
Stochastic dynamical systems approaches to climate dynamics



Henk A. Dijkstra, IMAU, Physics Department, Utrecht University, The Netherlands

NWO
Netherlands Organisation for Scientific Research

NESSC
NETHERLANDS EARTH SYSTEM SCIENCE CENTRE

IMAU
Institute for Marine and Atmospheric research Utrecht

Outline

1. Phenomena of Climate Variability

   - Time scales/patterns
   - Main questions

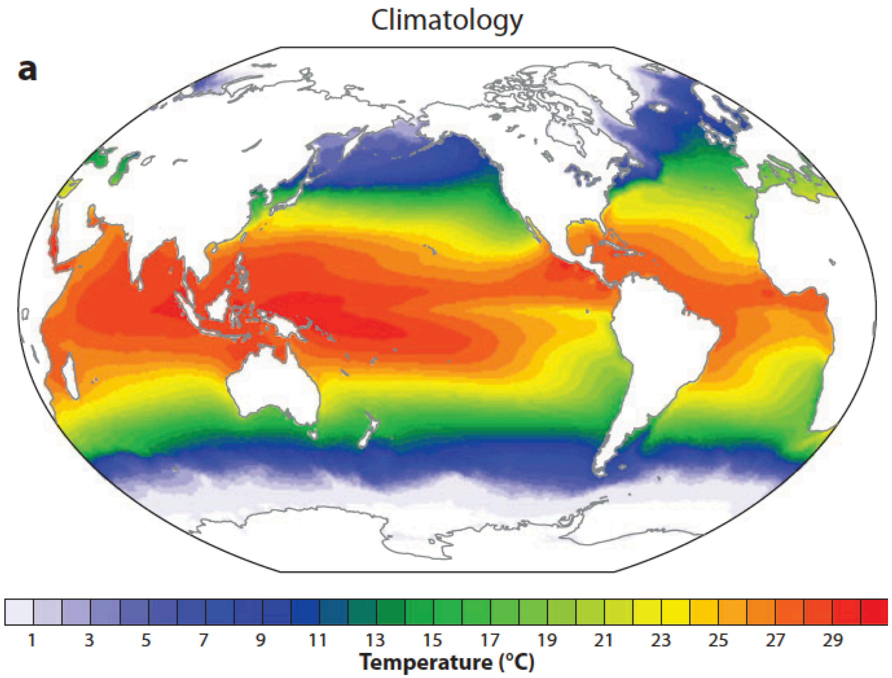
2. Dynamical systems approach

   - Model hierarchy
   - Concepts/techniques

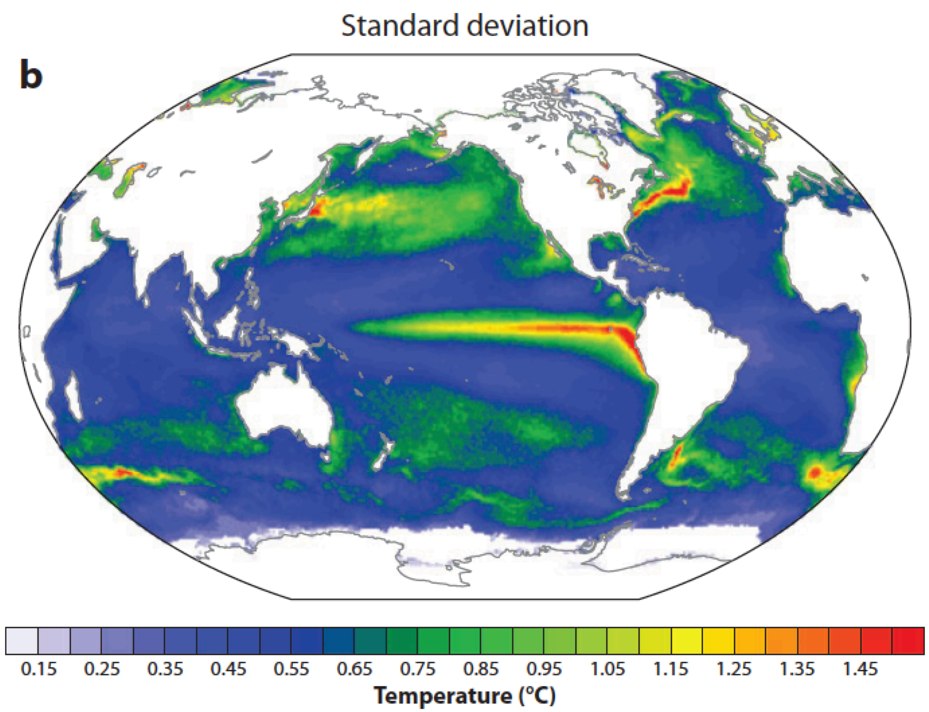
3. Specific examples

   - Wind-driven ocean gyre/jet flows
   - Zonal/blocked atmospheric flows

Sea surface temperature (SST) variability

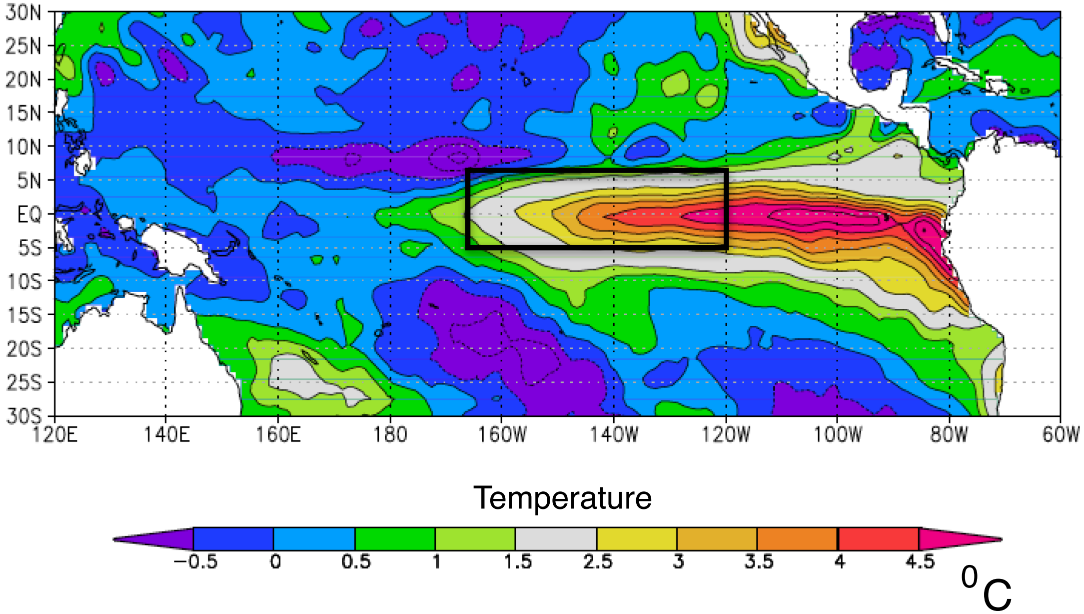


1982-2008

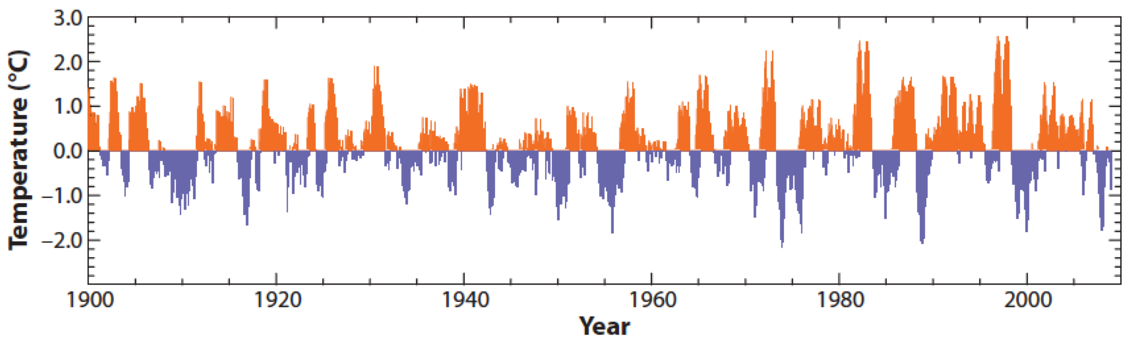


Deser et al., Ann. Rev. Mar. Sc. (2010)

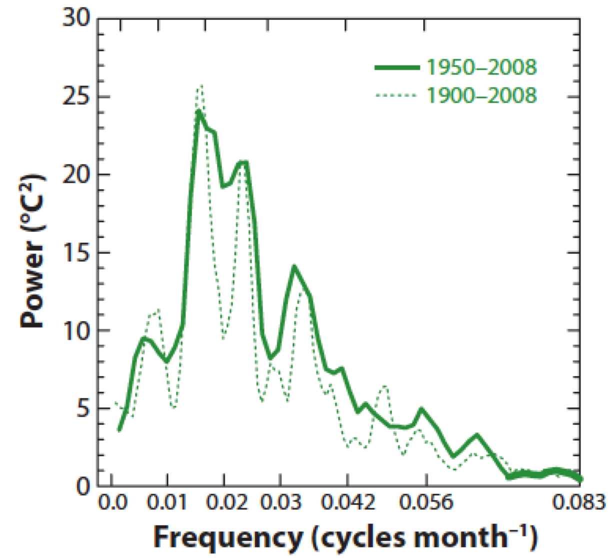
Patterns of variability: El Nino/Southern Oscillation (ENSO)



SST anomaly (1982-2010 mean) of December 1997

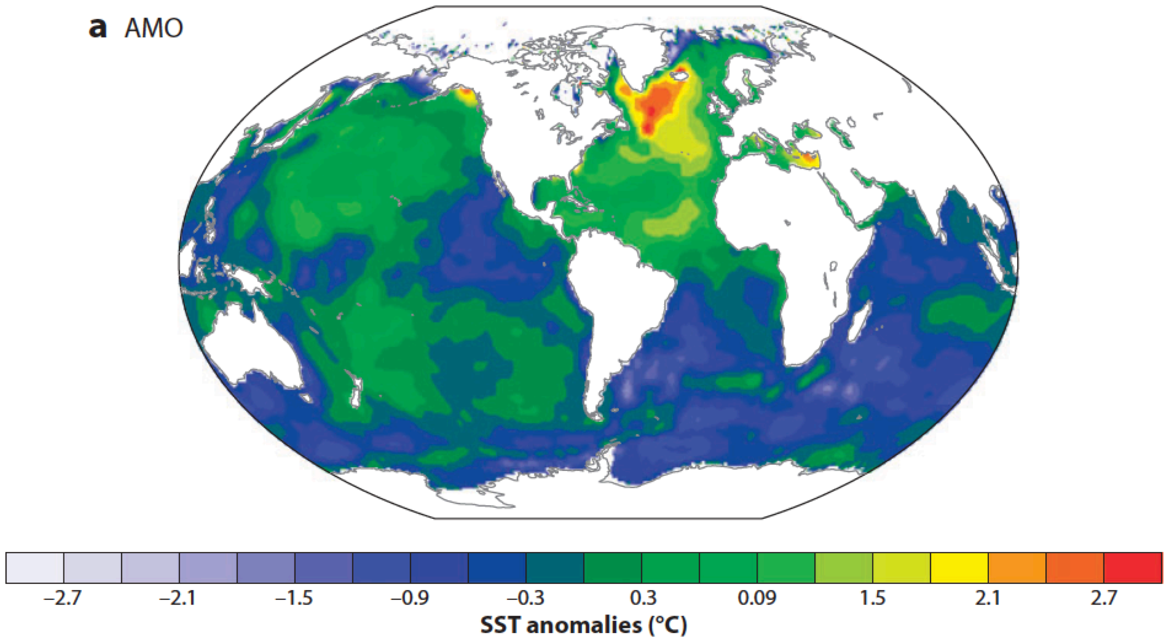


NINO3.4 index

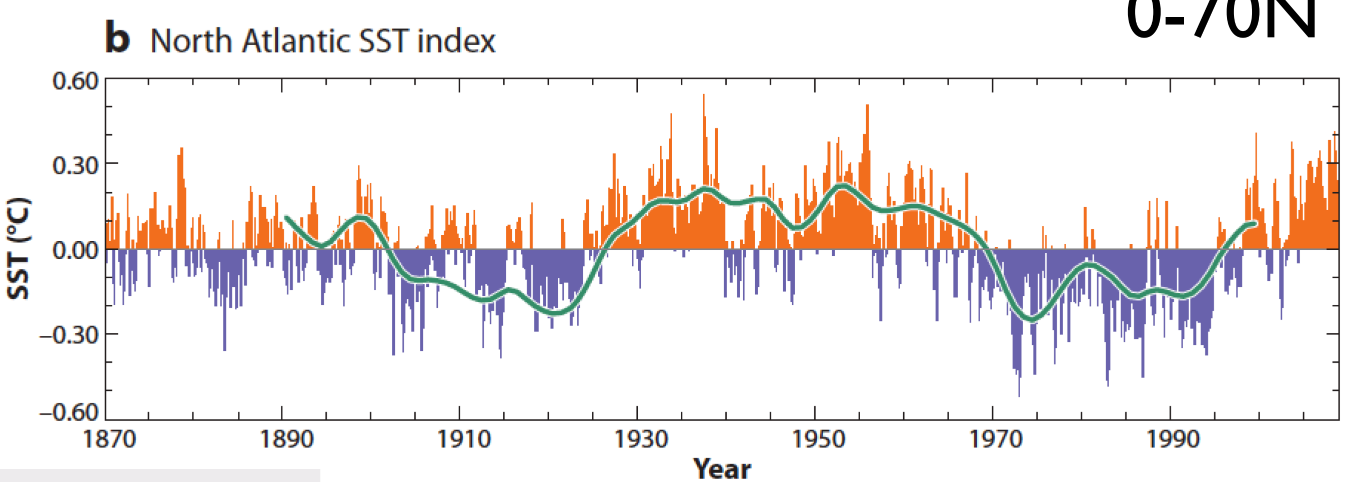


Deser et al., Ann. Rev. Mar. Sc. (2010)

Patterns of variability:
Atlantic Multidecadal Variability (AMV)

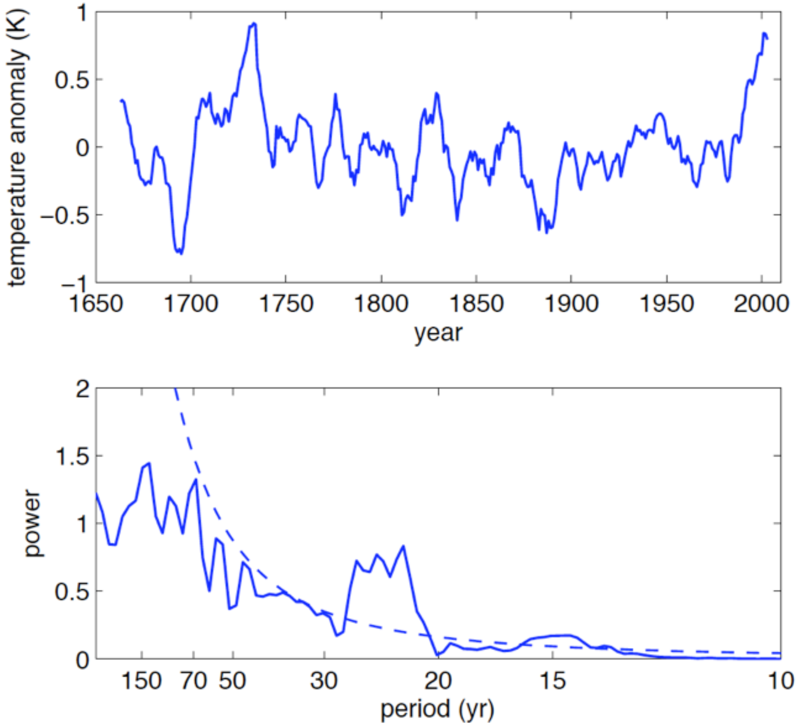


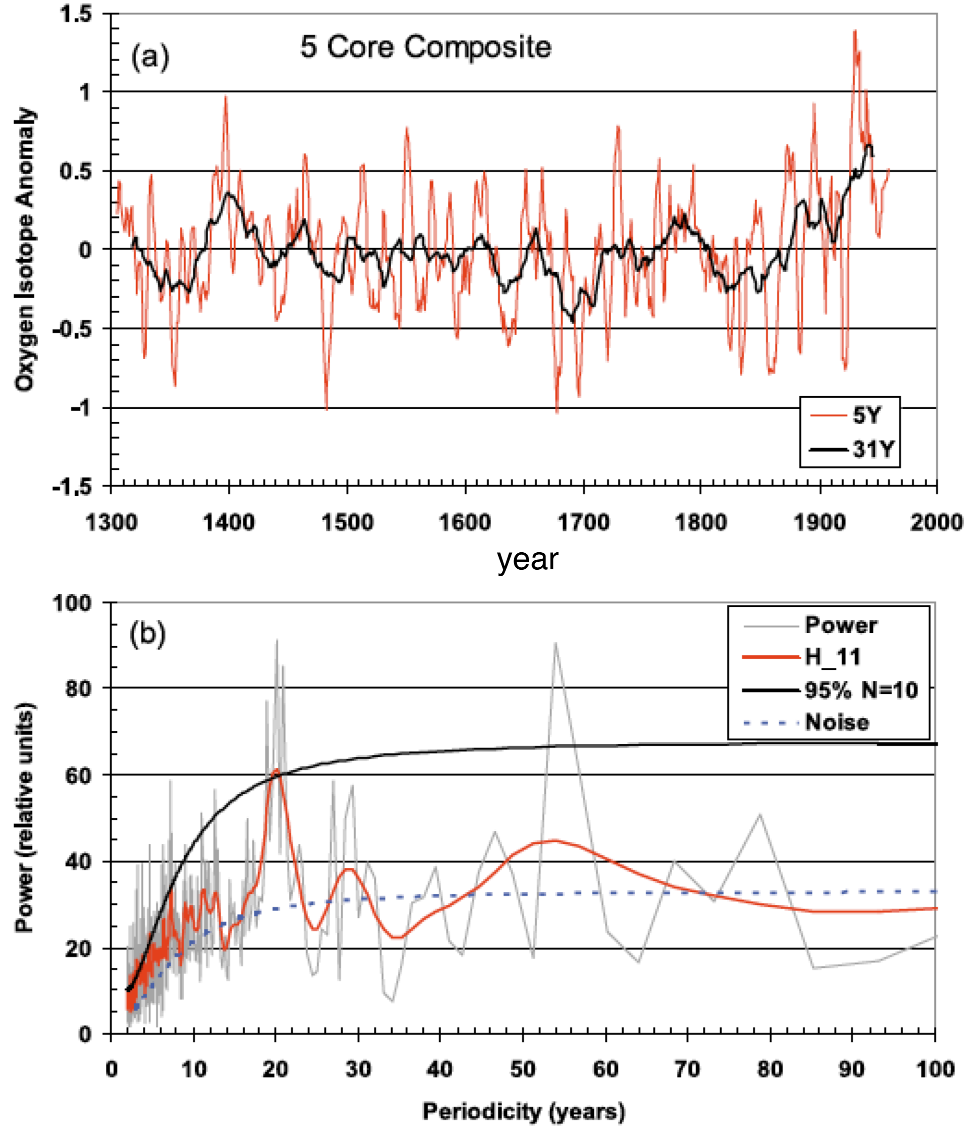
0-70N



Deser et al., Ann. Rev. Mar. Sc. (2010)

AMV: spectra





Central England Temperature

Greenland Ice Core

Chylek et al., GRL, (2011)

Main questions

Which processes determine the time scales and spatial patterns of these climate variability phenomena?

Ex: Why is the pattern of ENSO localized in the eastern Pacific and what sets its amplitude?

How does this variability interact with the background (e.g. longer time mean) climate state?

Ex: How does ENSO affect the global mean surface temperature?

Ex: What will be the change in ENSO behavior under global warming?

Intrinsic Variability

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) + \mathbf{g}_{ext}(t)$$

x: state vector

f: vector field

External (to Earth) forcing
(diurnal, seasonal, astronomical)

Reduction of state vector:

$$d\mathbf{X}_t = (\mathbf{F}(\mathbf{X}_t, t) + \mathbf{G}_{ext}(t) + \underline{\mathbf{H}(\mathbf{X}_t, t)})dt + \underline{\mathbf{h}(\mathbf{X}_t, t)d\mathbf{W}_t}$$

`Forcing'

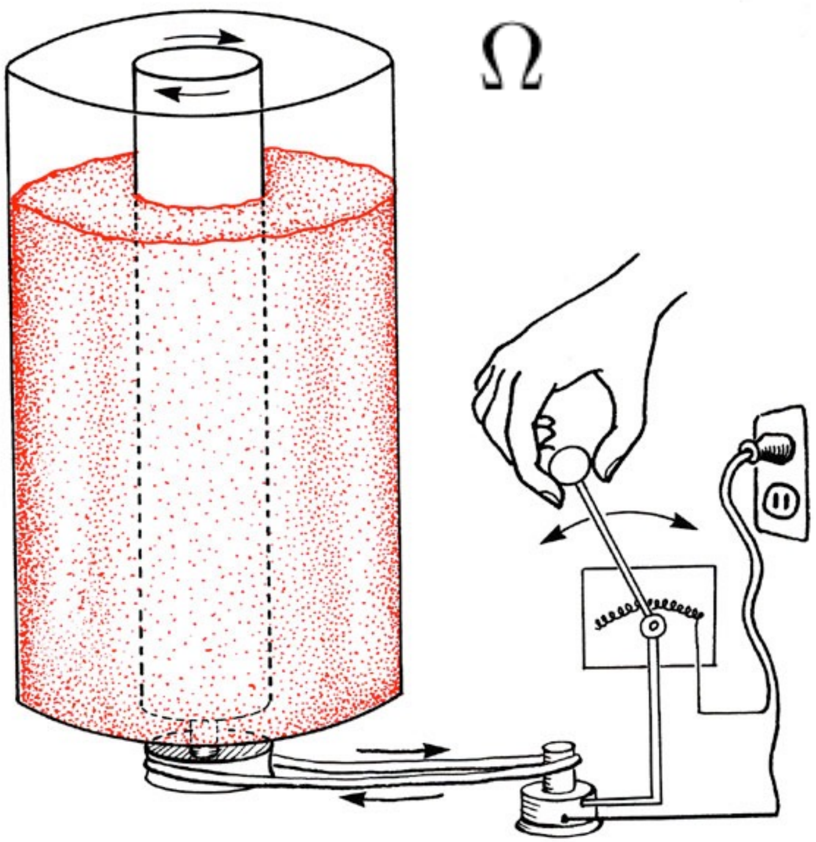
Representation
of unresolved processes

Intrinsic/internal variability: arises spontaneously through instabilities

Natural variability: combined variability due to external forcing and instabilities (without the anthropogenic `forcing')

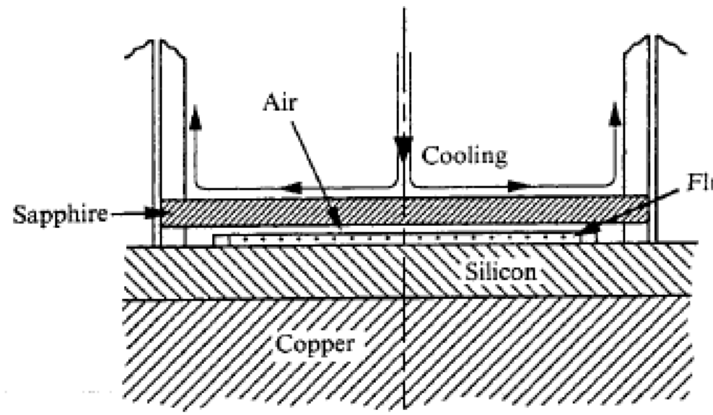
Dynamical systems approach

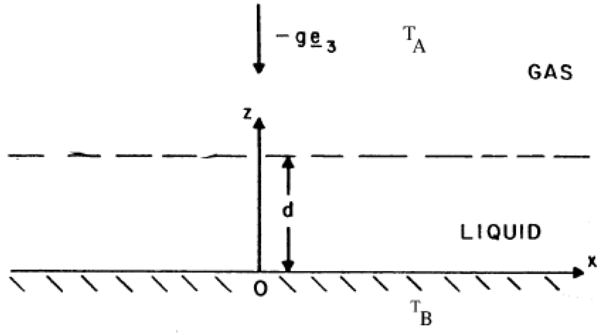
The Taylor-Couette Flow



Abraham, R. H. and Shaw, C. D., Dynamics, the Geometry of Behavior, (1988)

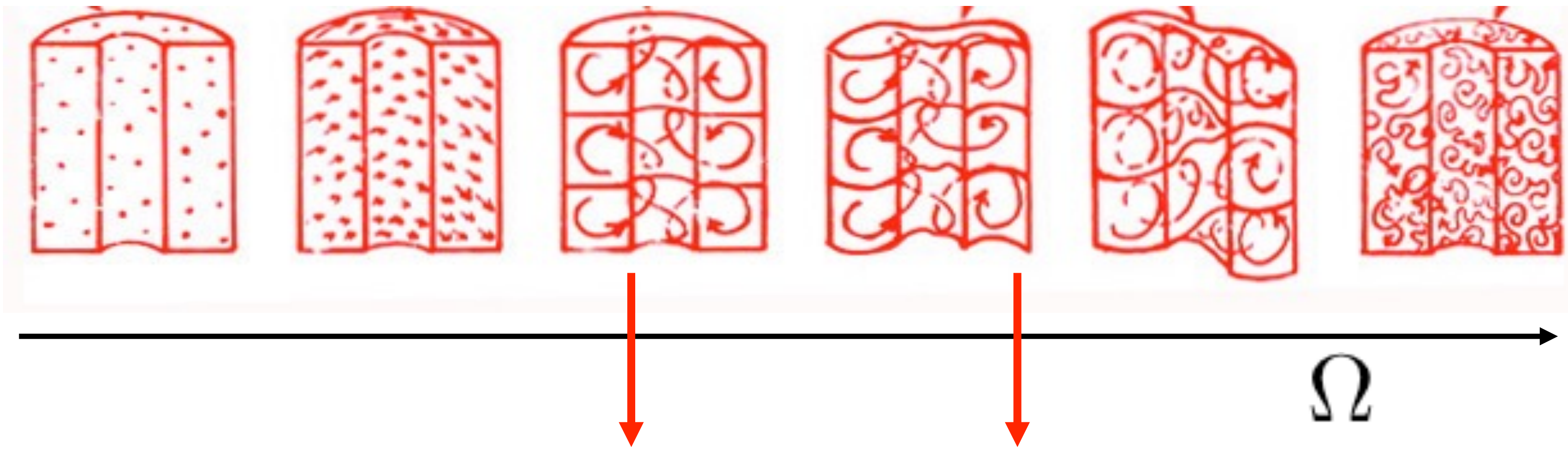
The Rayleigh-Benard Flow



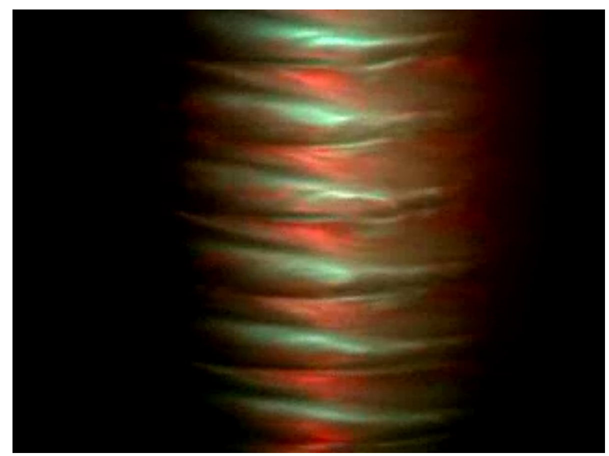


$$\Delta T = T_B - T_A$$

Transition behavior



Taylor vortices



Wavy vortices

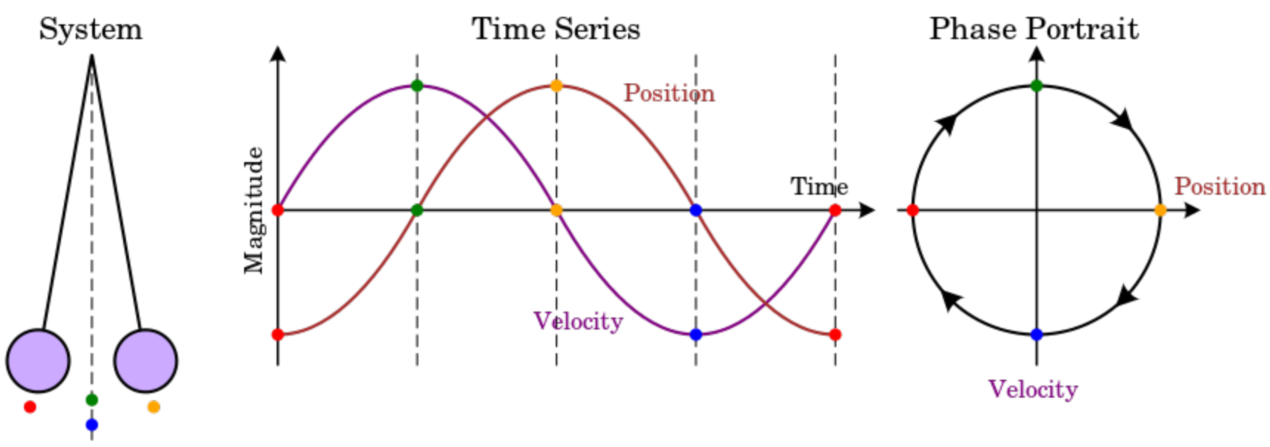
Phase space

$$x = \theta \quad ; \quad y = \frac{d\theta}{dt}$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -\frac{g}{L}x$$

degrees of freedom d = 2

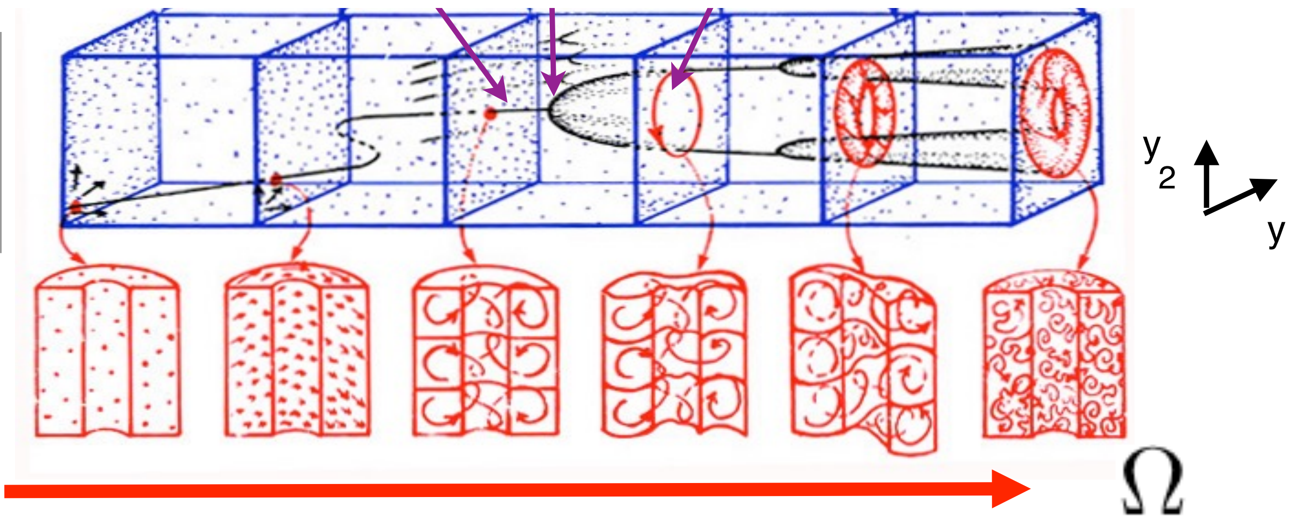


Geometry of motion!

Representations

Attractors in State/Phase-Parameter space

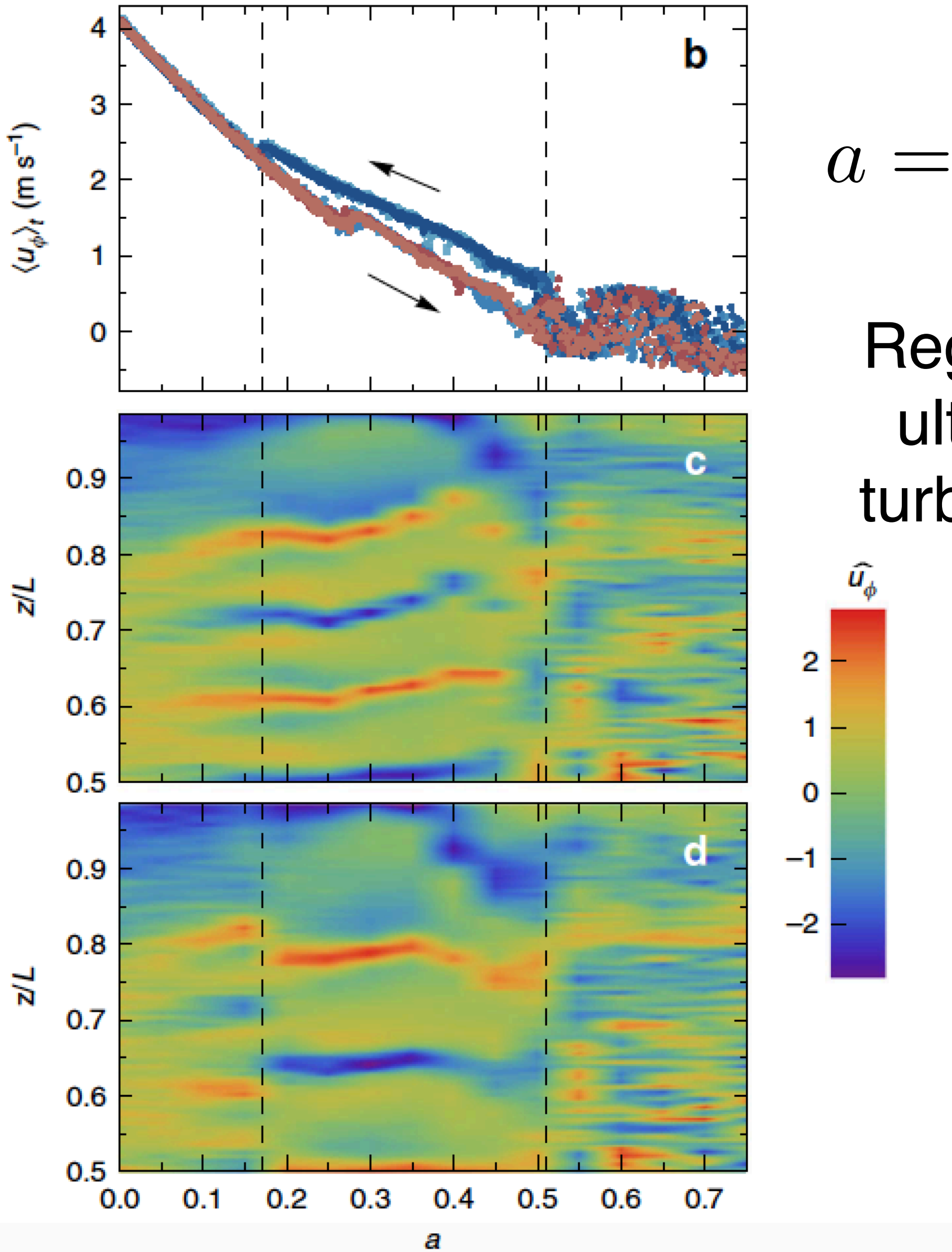
Physical space



Bifurcation Theory **Ergodic theory**

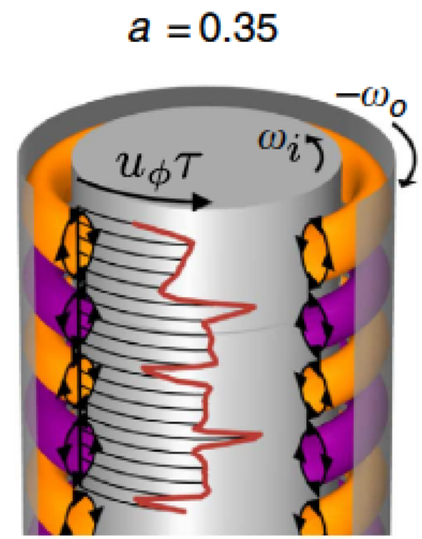
Steady --> Periodic --> Quasi-periodic --> … --> Irregular (Chaotic) … -> Turbulent

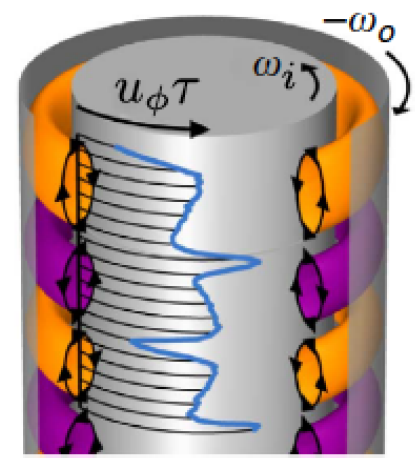
Multiple turbulent states



$$a = -\frac{\omega_0}{\omega_i}$$

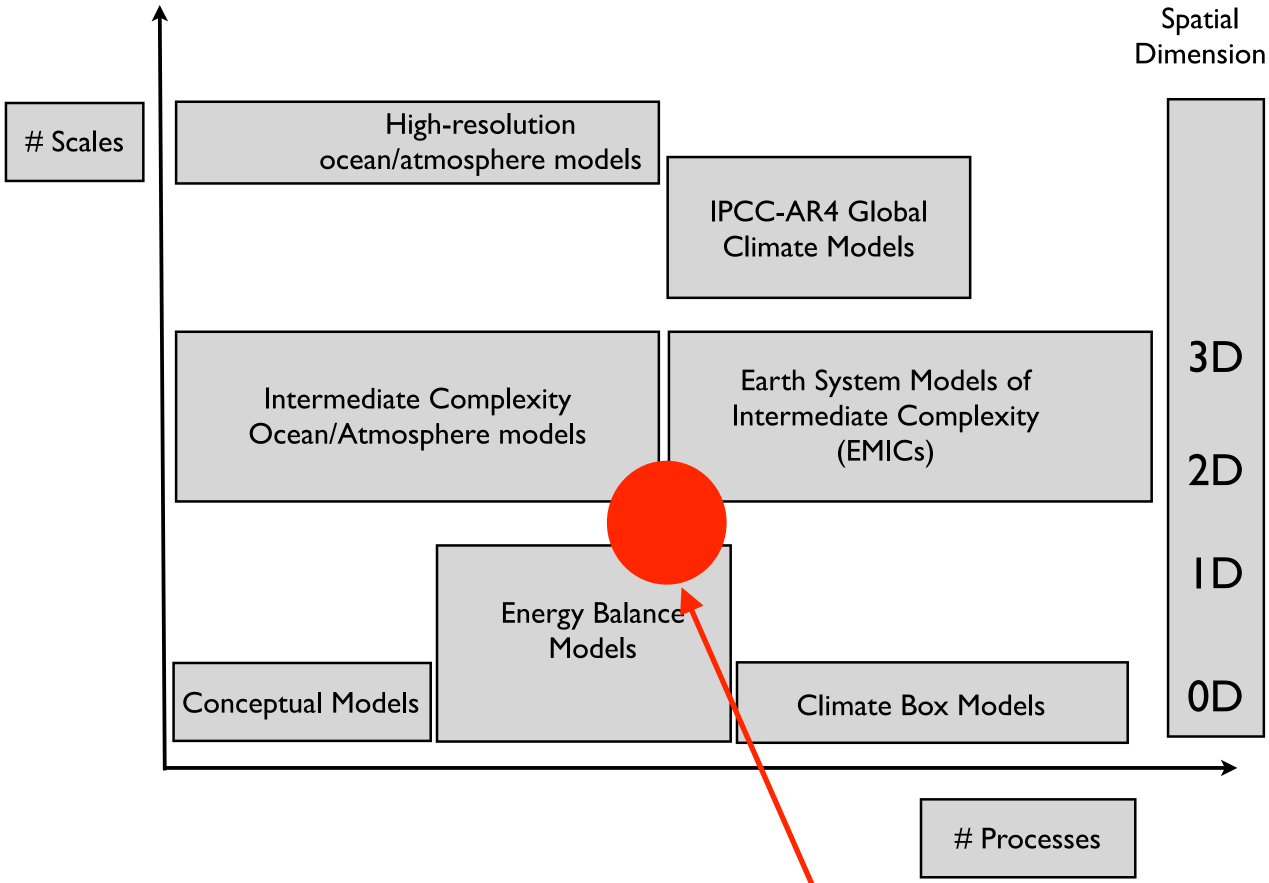
Regime of ultimate turbulence

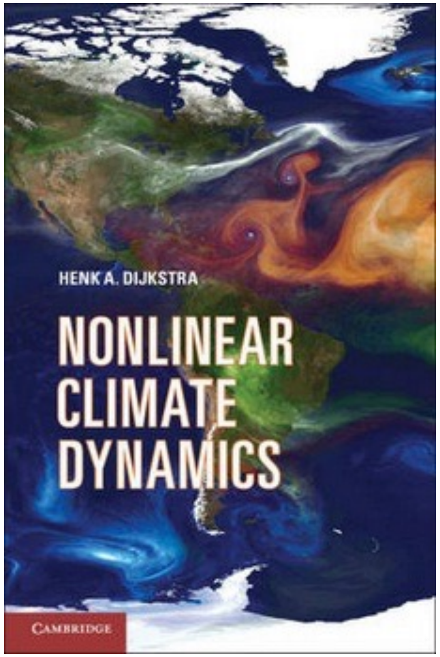




Huisman et al, Nature Comm, (2014)

Application to Climate Variability





chapter 6

‘Minimal’ Model:
‘just enough’ processes to capture phenomenon under study

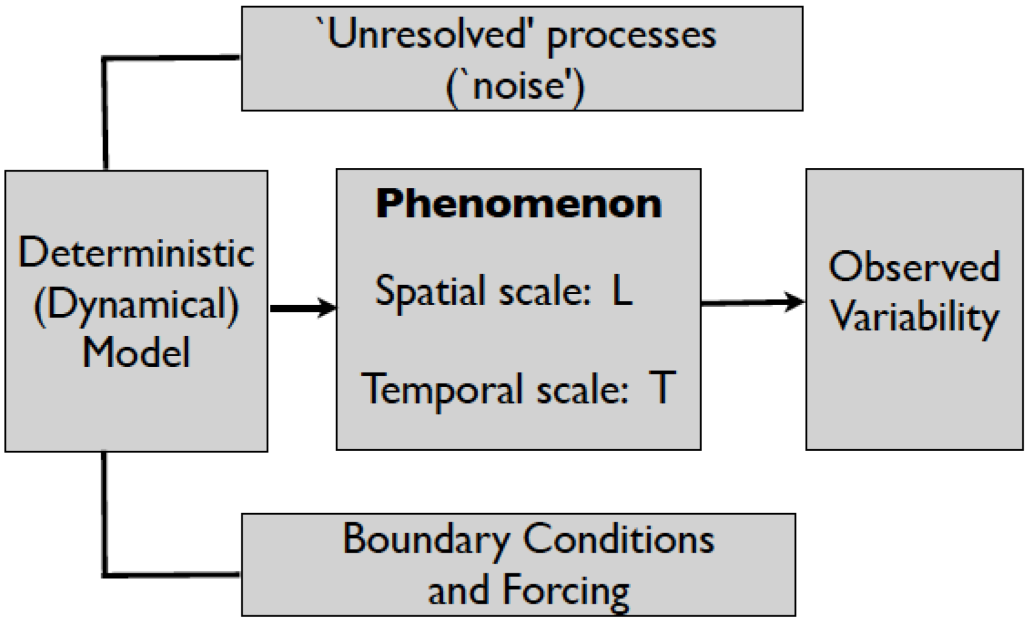
Stochastic dynamical systems approach

Hierarchy of models

Behavior (statistics) over the full parameter space

Geometrical view of motion

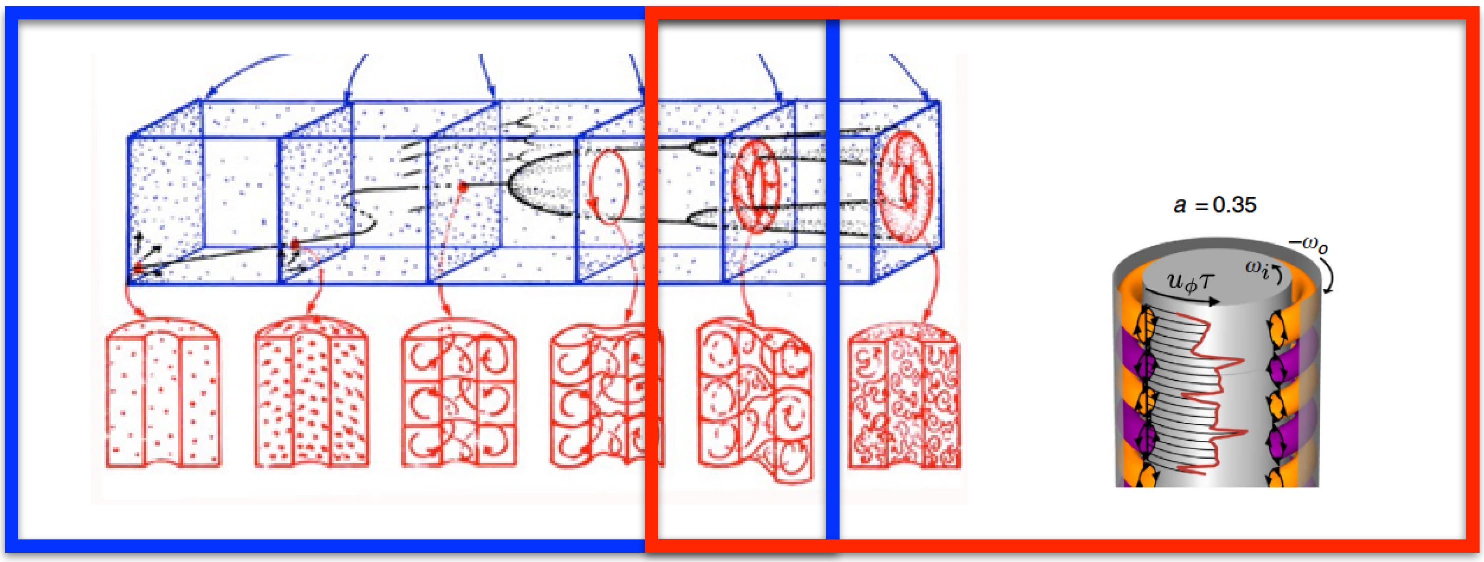
Dynamical system



Concepts/techniques

A. Bifurcation Theory

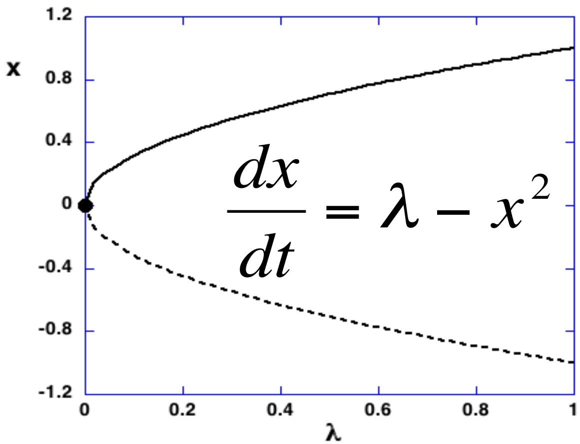
elementary transitions (pitchfork, saddle node, transcritical, Hopf)
normal modes (instability mechanisms), global bifurcations
transition to chaos, inertial manifolds, synchronization



B. Ergodic Theory

long time behavior of ensembles of trajectories
invariant measures, transfer operators
evolution of correlations

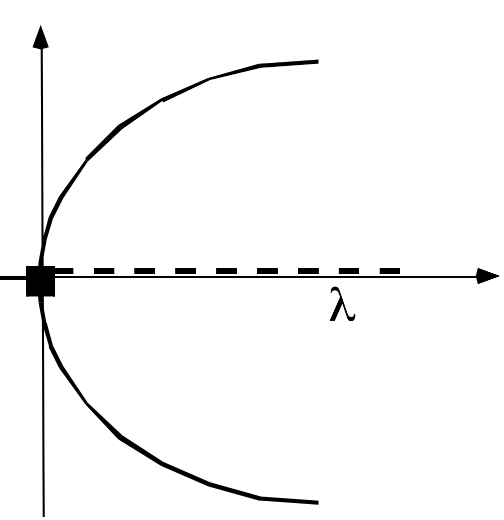
Elementary transitions (co-dim 1 bifurcations)



$$\frac{dx}{dt} = \lambda - x^2$$

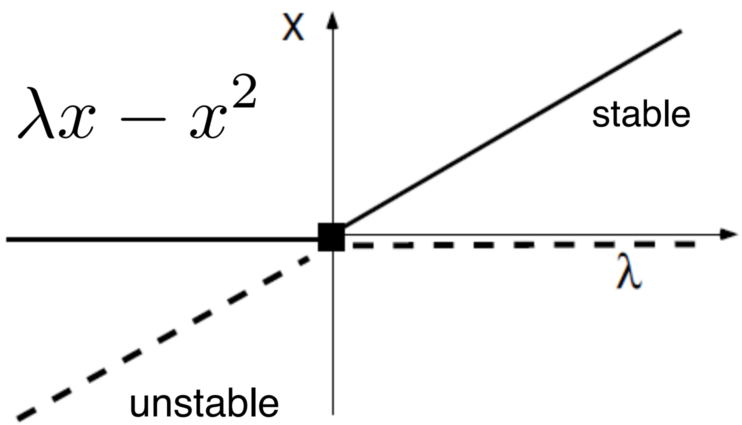
Saddle-node (non-existence)

$$\frac{dx}{dt} = \lambda x - x^3$$



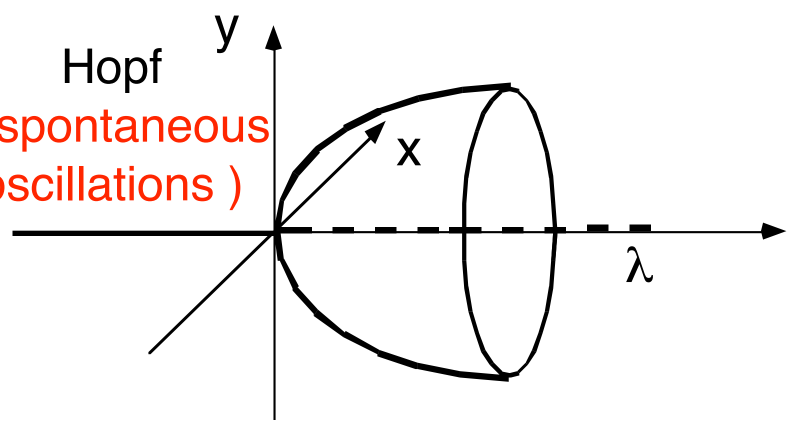
Pitchfork
(symmetry-breaking)

$$\frac{dx}{dt} = \lambda x - x^2$$



Transcritical (exchange of stability)

Hopf
(spontaneous oscillations)

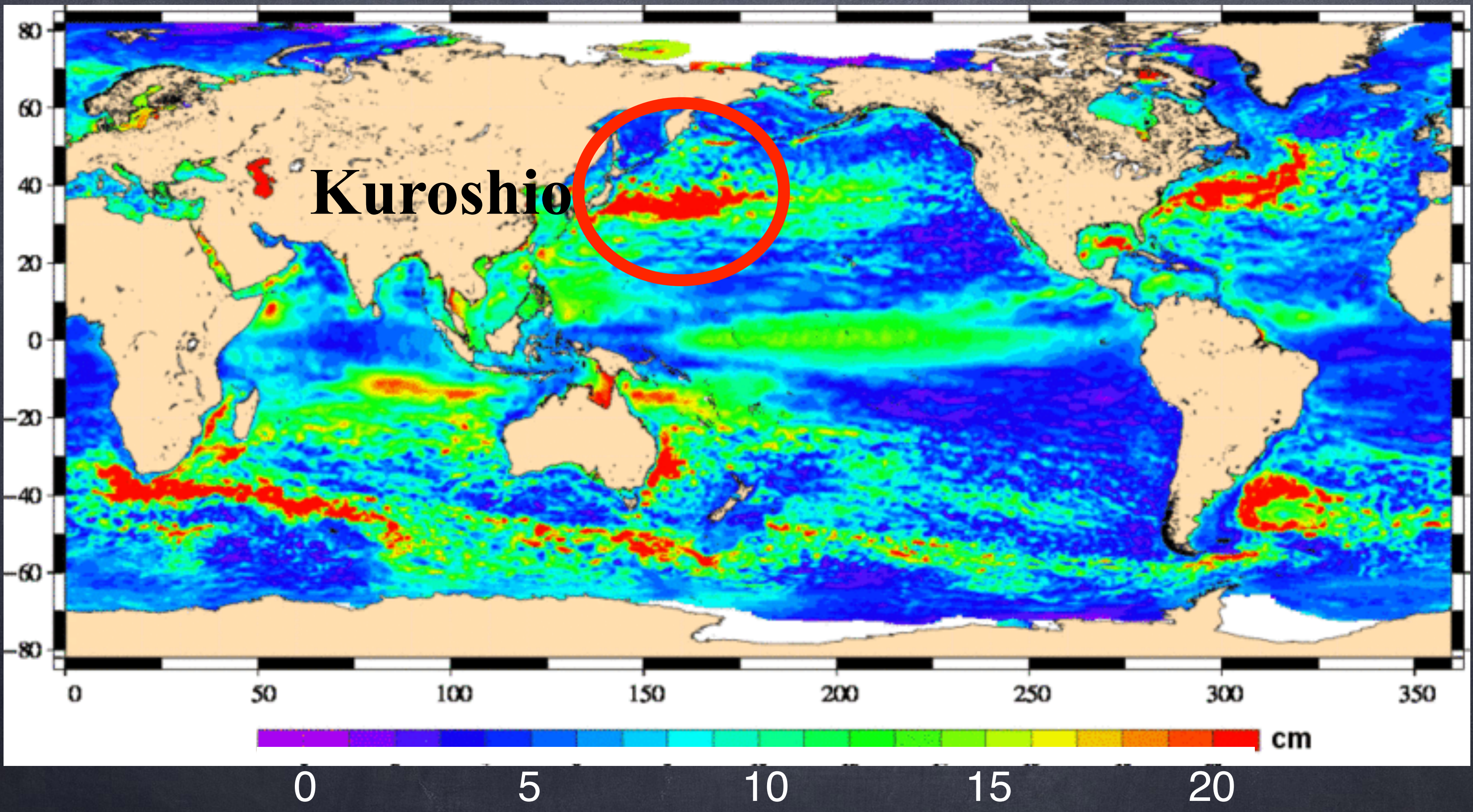


$$\dot{x} = \lambda x - \omega y - x(x^2 + y^2)$$

$$\dot{y} = \lambda y + \omega x - y(x^2 + y^2)$$

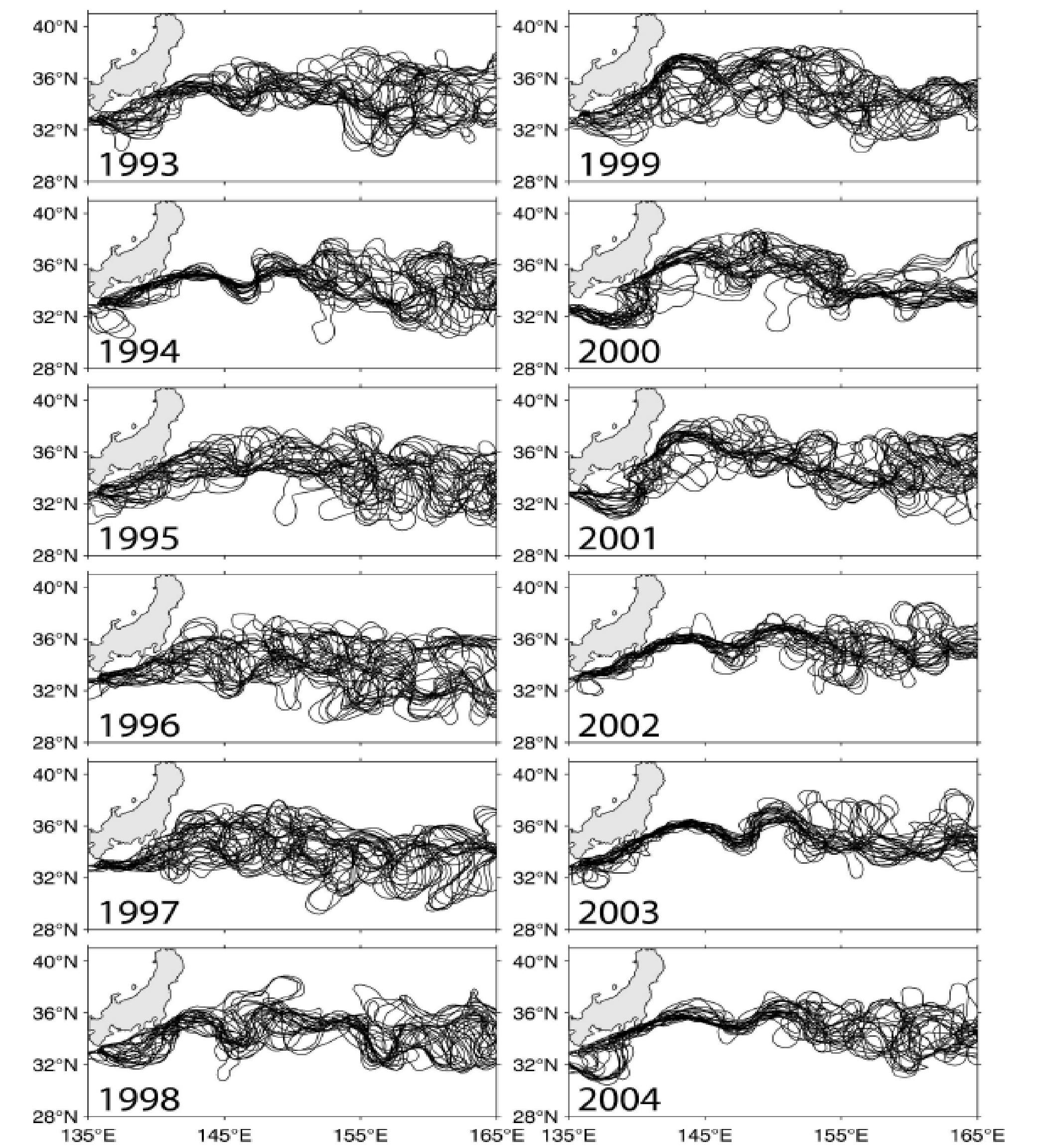
Hopf bifurcation (flutter)

Example 1: bifurcation theory
Ocean western boundary current variability



RMS of sea surface height (SSH) variability

Kuroshio path: 1993-2004

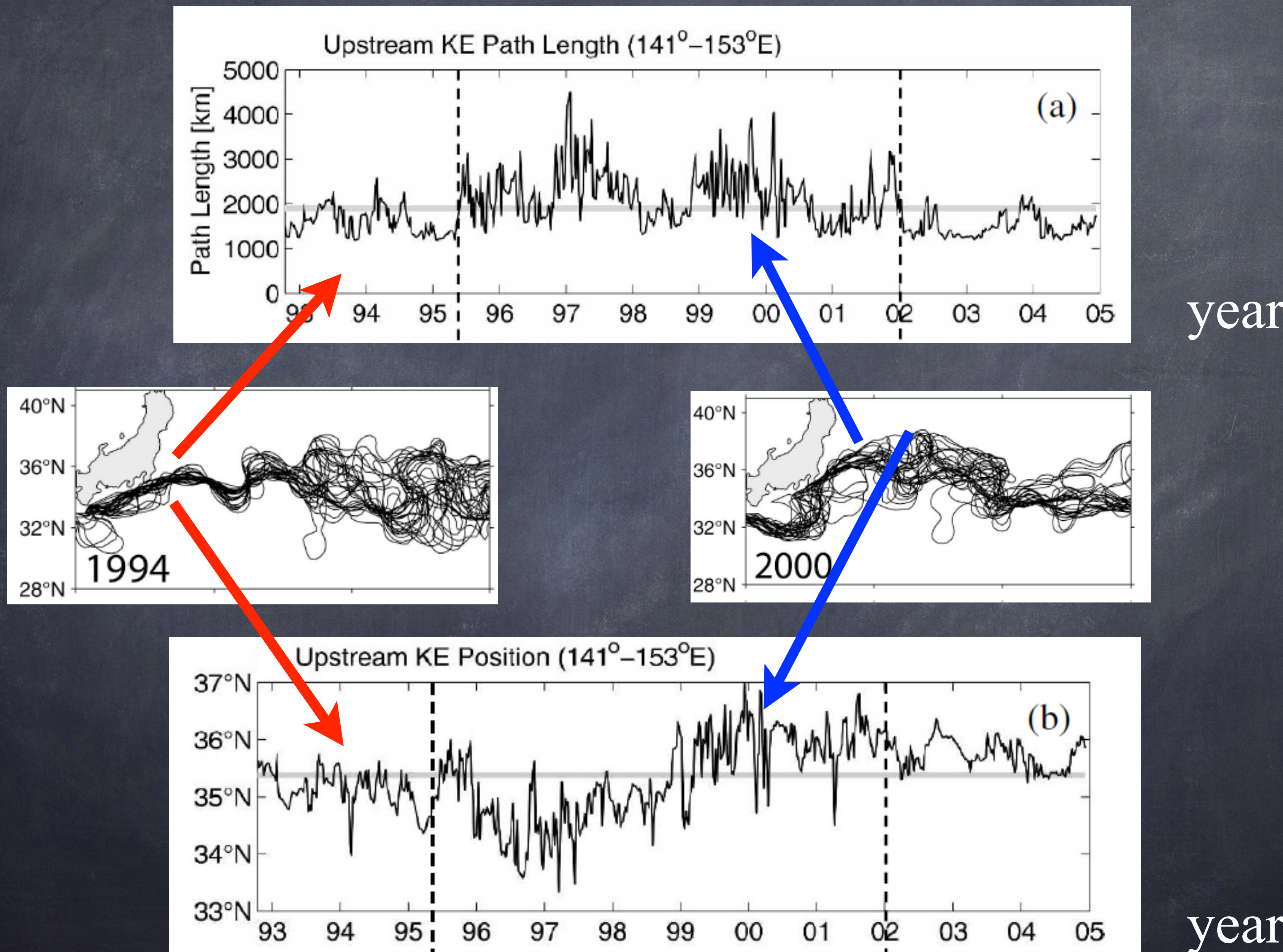


Bi-weekly mean Kuroshio path from altimetry (170 cm sea level contour)

Interannual-decadal time scale transitions between different Kuroshio paths.

Qiu and Chen, JPO, 2005

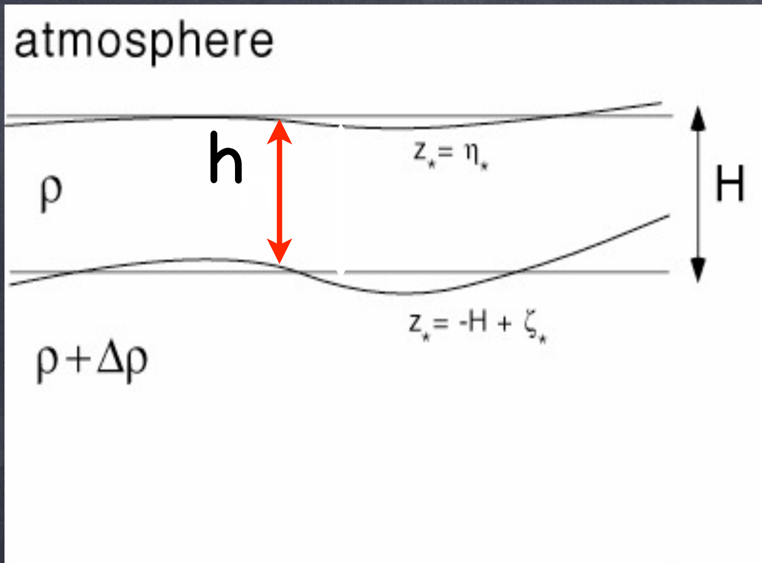
SSH metrics

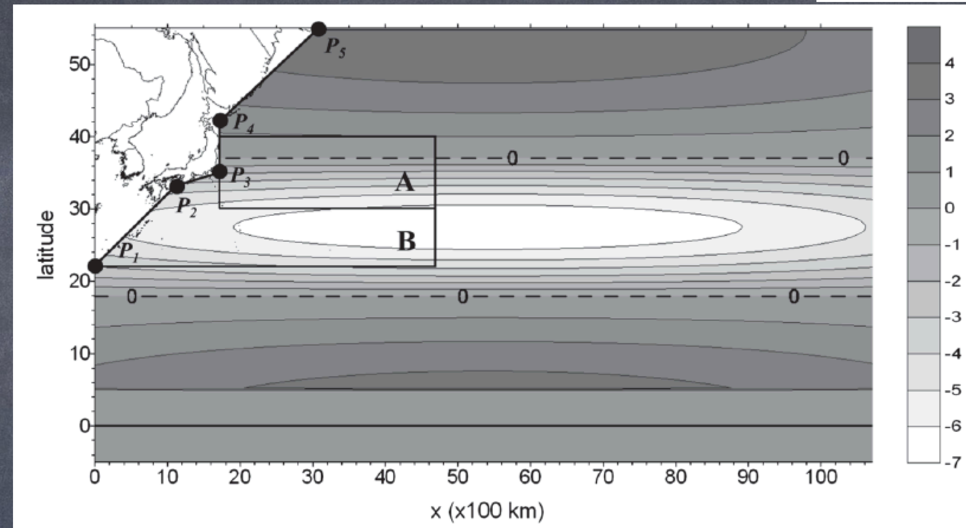


Upstream KE Path Length (141°–153°E)
Path Length [km]
5000
4000
3000
2000
1000
0
(a)
93
94
95
96
97
98
99
00
01
02
03
04
05
year
40°N
36°N
32°N
28°N
1994
2000
Upstream KE Position (141°–153°E)
37°N
36°N
35°N
34°N
33°N
(b)
year

A ‘minimal’ model

$10^{-8}\,\mathrm{N\,m^{-3}}$





reduced gravity shallow-water model

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + f\mathbf{k}\times\mathbf{u} = -g'\nabla\eta + A_H\nabla^2\mathbf{u} + \frac{\boldsymbol{\tau}}{\rho h} - \gamma\mathbf{u}|\mathbf{u}| \quad \text{and}$$

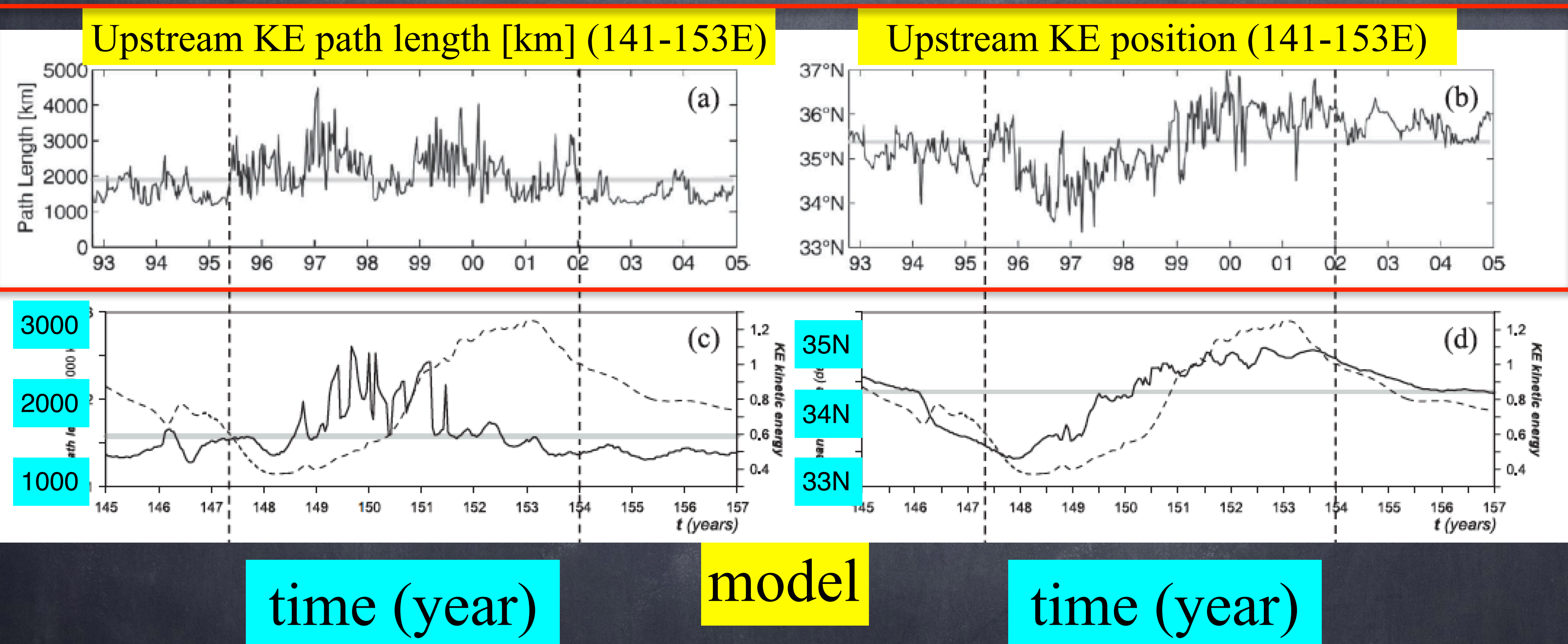
$$\frac{\partial h}{\partial t} + \nabla\cdot(h\mathbf{u}) = 0,$$

steady wind stress

control parameter:
`lateral friction’

20 km resolution ⟶ d ~ 500,000

Model-Data comparison: SSH metrics

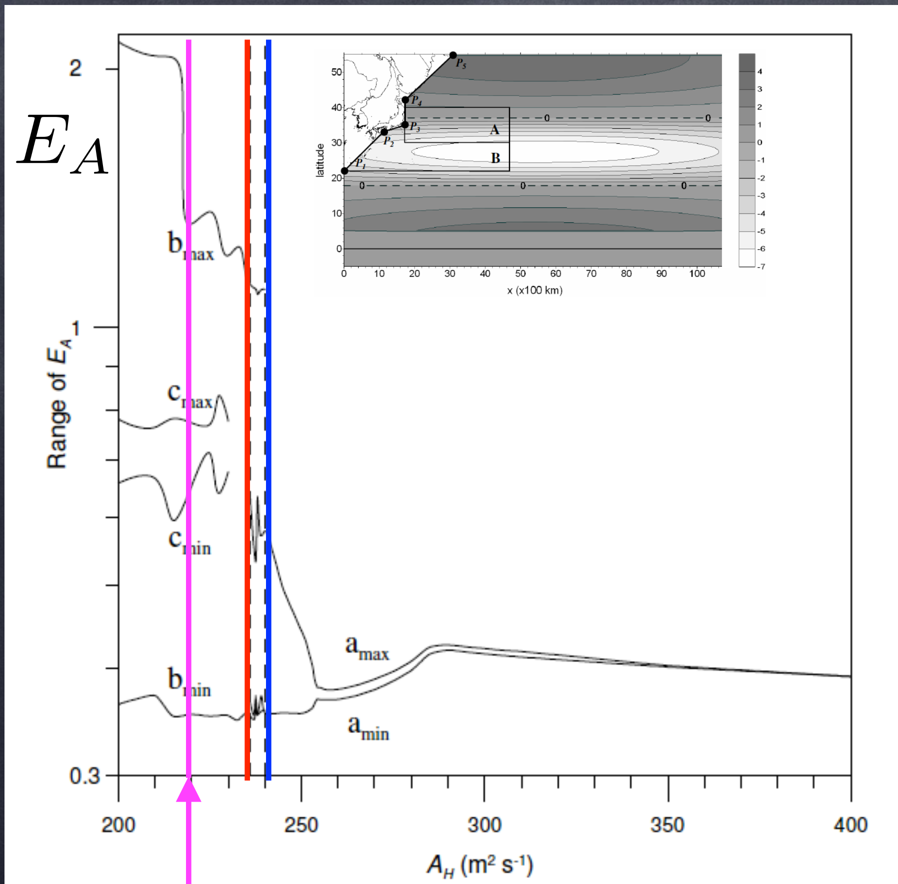


$$A_H = 220\ m^2/s$$

Pierini et al., JPO, (2009)

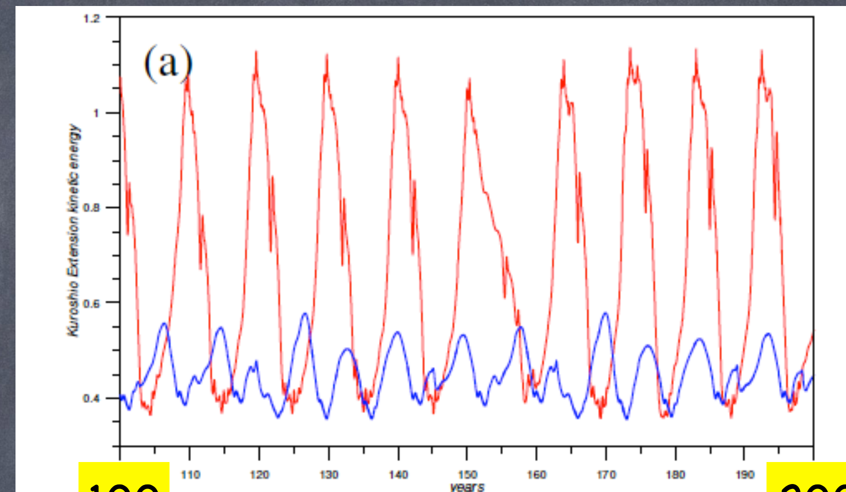
Lateral mixing variation: transition behavior!

E_A



Model-observation comparison

E_A

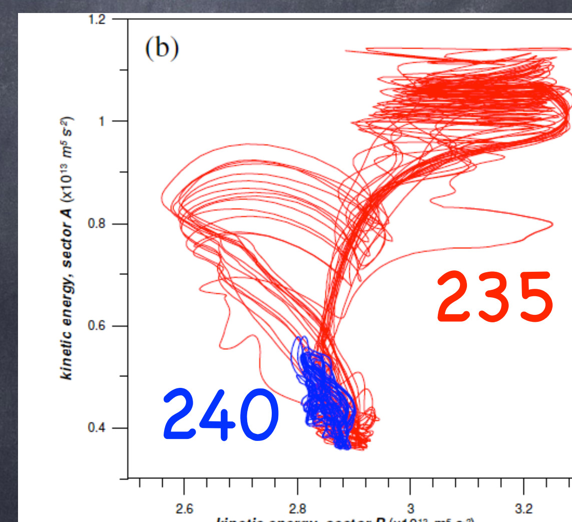


100

200

$time(yr)$

E_A



235

240

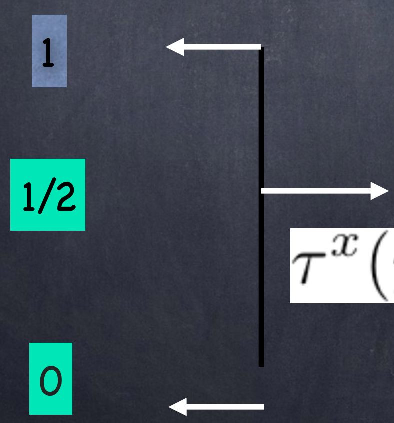
E_B

Bifurcation diagram
Quasi-geostrophic (QG) barotropic model

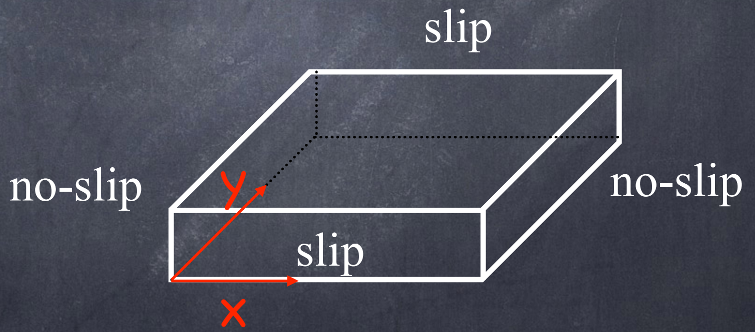
$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + \beta\frac{\partial \psi}{\partial x} = \frac{1}{Re}\nabla^2\zeta + \alpha\left(\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}\right)$$

$u = \frac{\partial \psi}{\partial y}$; $v = -\frac{\partial \psi}{\partial x}$; $\zeta = \nabla^2\psi$

Double-gyre wind stress:

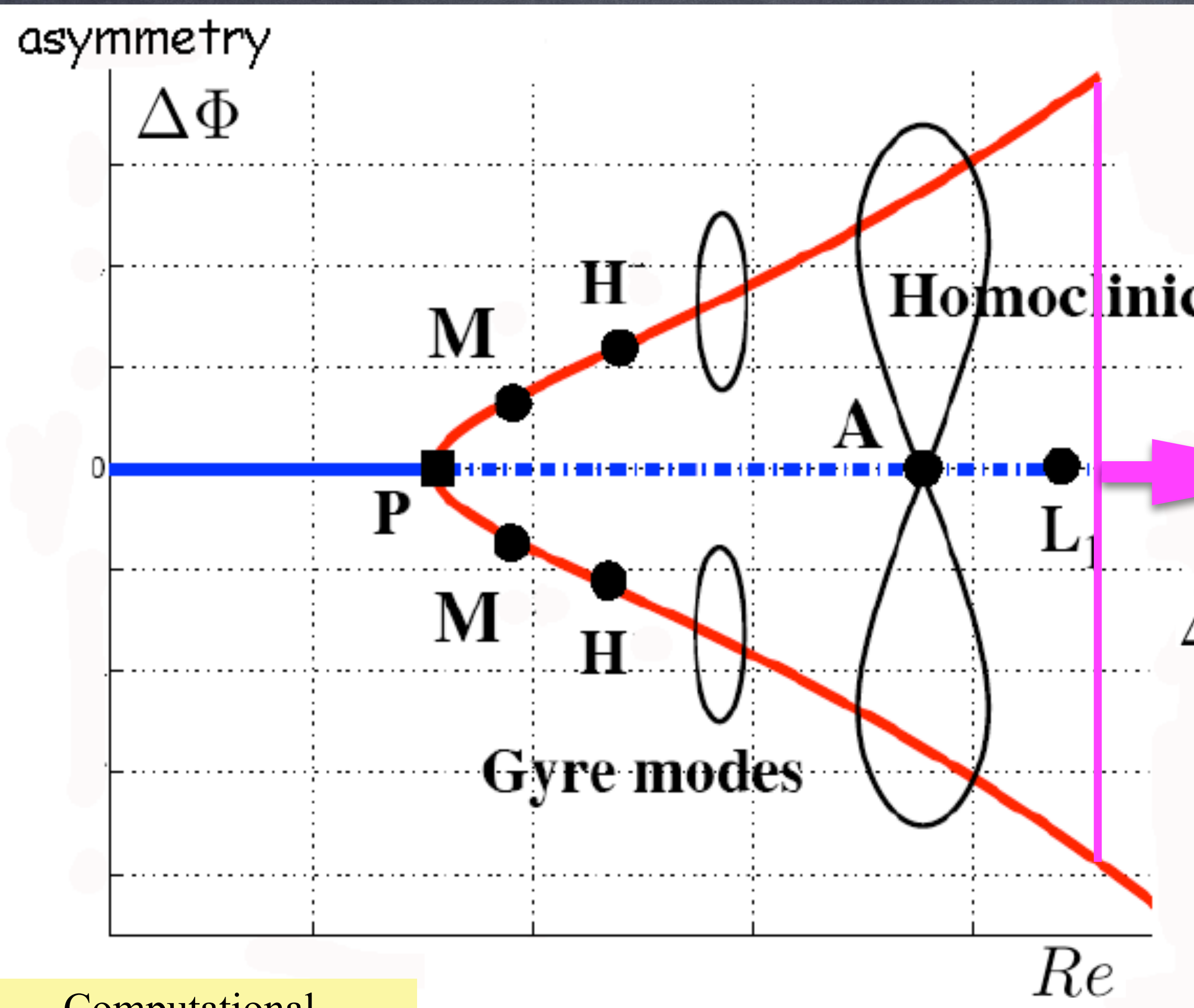


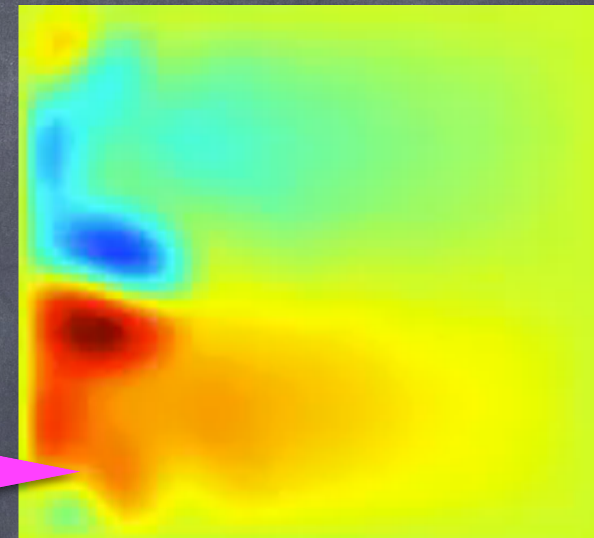
$$\tau^x(y) = -\cos\ 2\pi y$$

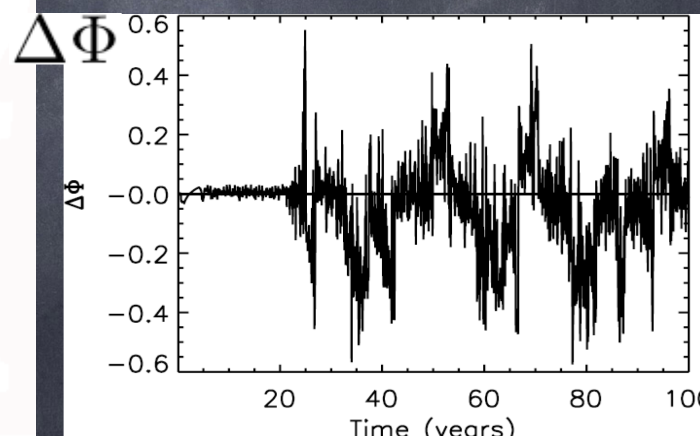


Control parameter: $\mathrm{Re} = \frac{UL}{A_H}$

Bifurcation diagram QG-model: schematic







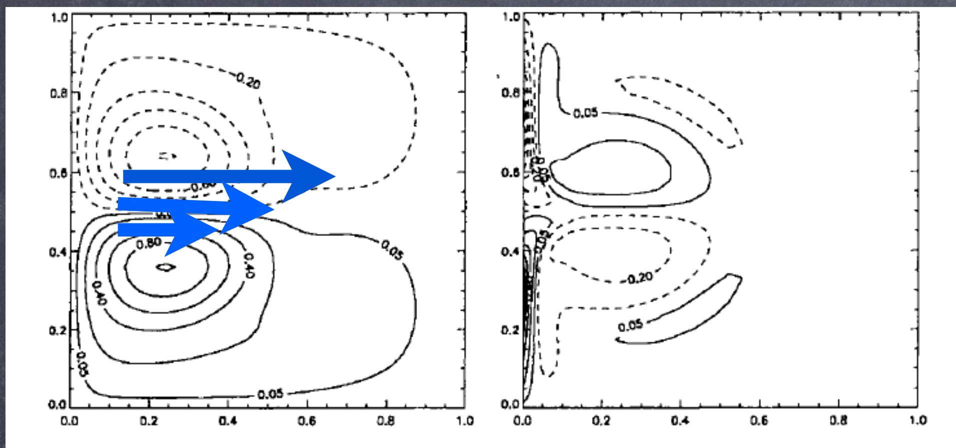
Computational techniques: Pseudo-arclength continuation

$$\Delta\Phi = \frac{\psi_{min} + \psi_{max}}{|\ \psi_{max}\ |}$$

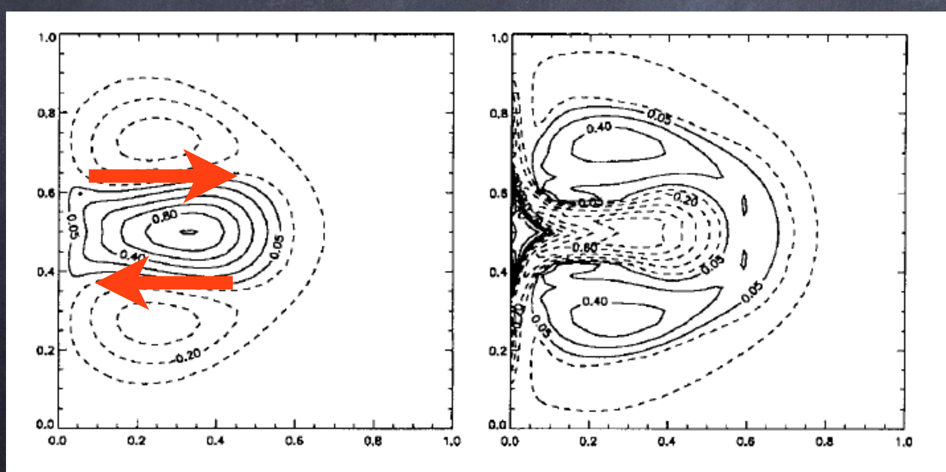
Simonnet et al., JMR, (2005)

Symmetry breaking: shear instability at first pitchfork

Streamfunction Vorticity



Steady state at pitchfork



Normal (P) mode at Pitchfork

Details: Derive low-order Model

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + \beta\frac{\partial \psi}{\partial x} = \frac{1}{Re}\nabla^2\zeta + \alpha\left(\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}\right) \quad -\mu\zeta$$

$$\zeta = \nabla^2\psi$$

Choose wind-stress strength as control parameter

$$\begin{aligned}
\psi(x,y,t) &= A_1(t)G(x)\sin y + A_2(t)G(x)\sin 2y + \\
&+ A_3(t)G(x)\sin 3y + A_4(t)G(x)\sin 4y \\
G(x) &= e^{-sx}\sin x \qquad x\in[0,\pi], y\in[0,\pi]
\end{aligned}$$

+ Galerkin projection gives:

Simonnet et al., JMR, (2005)

Results: Low-order model

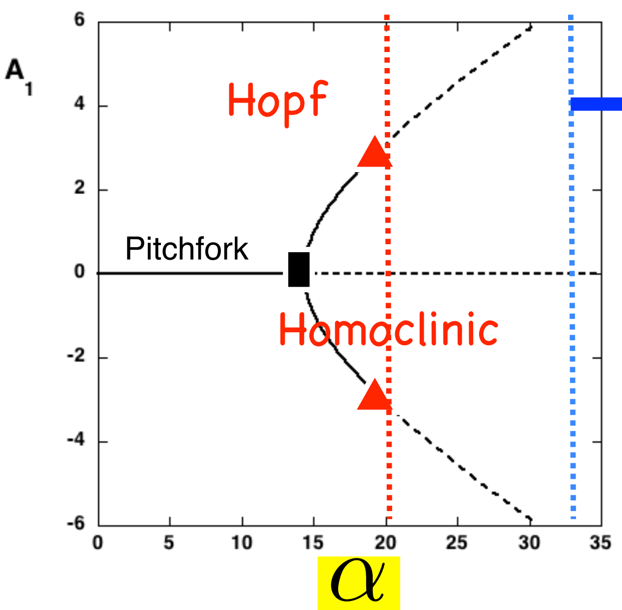
$$
\begin{aligned}
\frac{dA_1}{dt} &= c_1(A_1A_2 + A_2A_3 + A_3A_4) - A_1 \\
\frac{dA_2}{dt} &= 2c_2(A_1A_3 + A_2A_4) - c_2A_1^2 - A_2 + c_5\alpha \\
\frac{dA_3}{dt} &= c_3A_1(A_4 - A_2) - A_3 \\
\frac{dA_4}{dt} &= -c_4A_2^2 - 2c_4A_1A_3 - A_4
\end{aligned}
$$

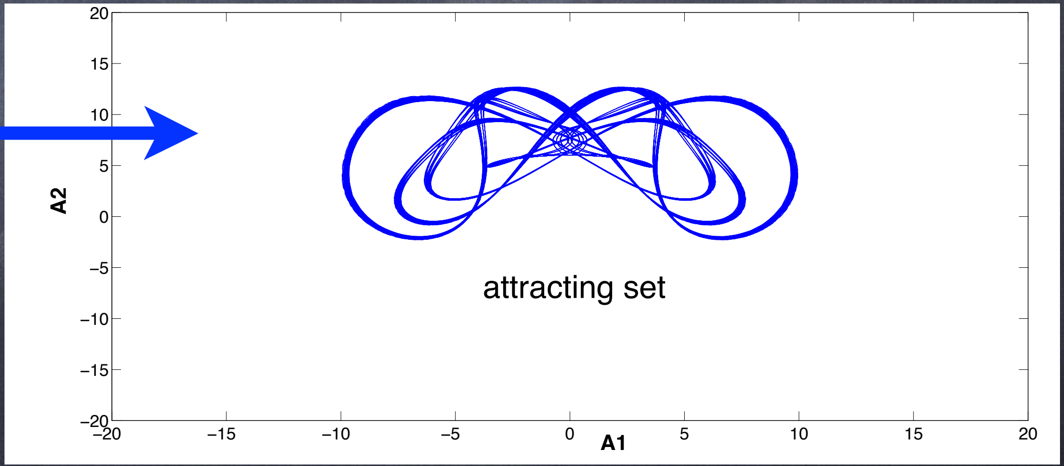
Wind-stress amplitude

$$\tau_0 = 0.1\ Pa$$

$$\rightarrow \alpha = 20$$

AUTO/MatCont



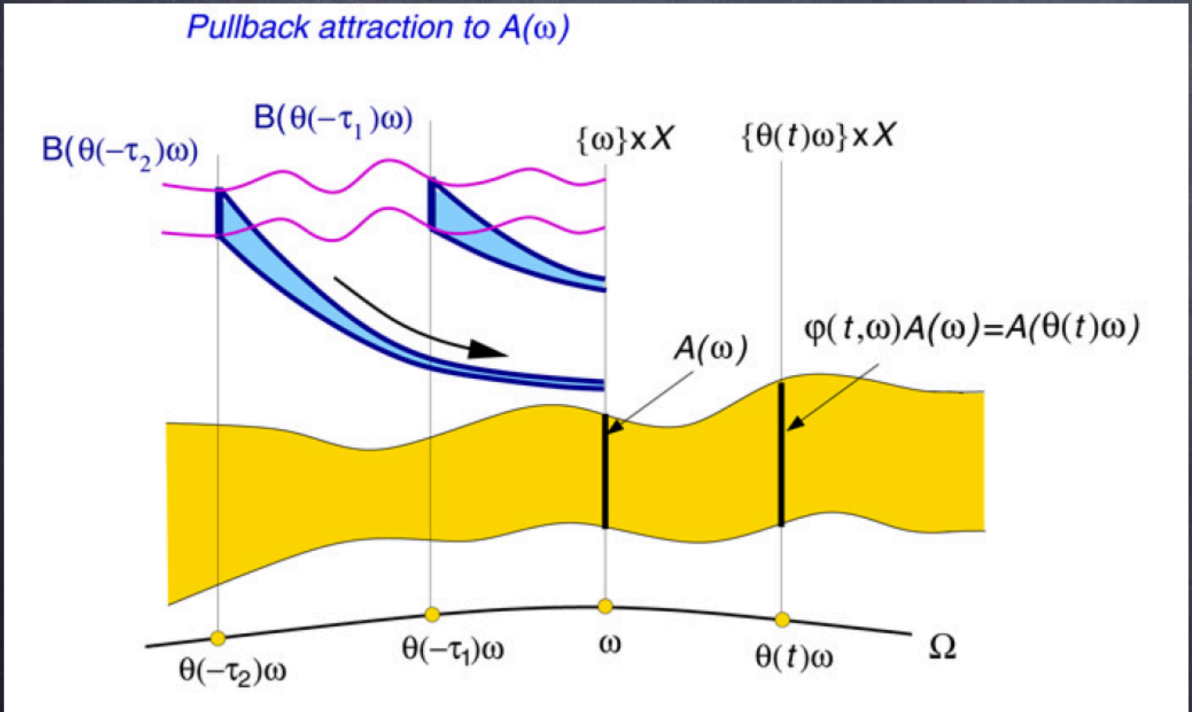


Lyapunov exponent: $\lambda_1 > 0$

Wind-stress noise in low-order model

SDE

$$
\begin{aligned}
dA_1 &= (c_1(A_1A_2 + A_2A_3 + A_3A_4) - A_1)dt \\
dA_2 &= (2c_2(A_1A_3 + A_2A_4) - c_2A_1^2 - A_2)dt + c_5\alpha(dt + \sigma \circ dW_t) \\
dA_3 &= (c_3A_1(A_4 - A_2) - A_3)dt \\
dA_4 &= (-c_4A_2^2 - 2c_4A_1A_3 - A_4)dt
\end{aligned}
$$

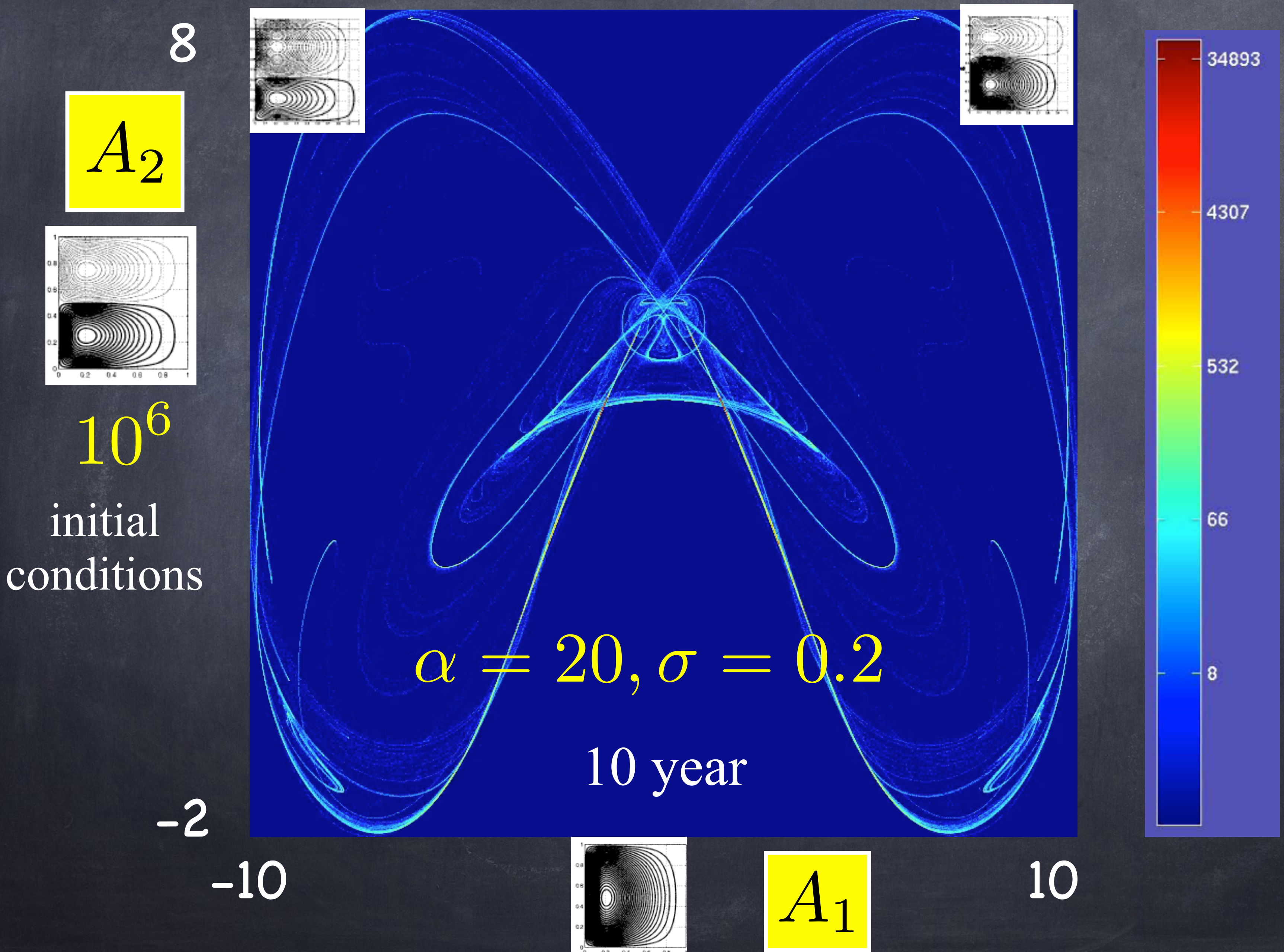


Computation of sample measures

single noise realization

Chekroun, Simonnet, Ghil Physica D, (2008)

Results: sample measure



Effects of noise in the wind-stress forcing on intrinsic variability in PDE models?

1. Local PDF through linearized dynamics

Kuehn, SIAM J. Sci. Comp., (2012)

2. Dynamical Orthogonal Field theory

Sapsis and Lermusiaux, Physica D, (2009)

Sapsis and Majda, Physica D, (2013)

3. Non-Markovian model reduction techniques

Chekroun et al., Springer, (2015)

Dynamically Orthogonal Field Equations, I

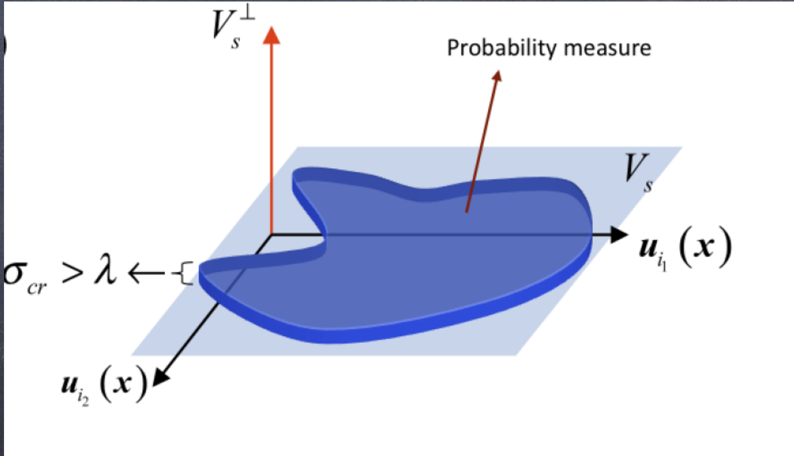
General SPDE

$$\frac{\partial \mathbf{u}(\mathbf{x},t;\omega)}{\partial t} = \mathcal{L}\left[\mathbf{u}(\mathbf{x},t;\omega);\omega\right], \quad \mathbf{x} \in D, \quad t \in \mathcal{T}, \quad \omega \in \Omega$$

Karhunen-Loeve

$$\mathbf{u}(\mathbf{x},t;\omega) = \bar{\mathbf{u}}(\mathbf{x},t) + \sum_{i=1}^{s} Y_i(t;\omega)\,\mathbf{u}_i(\mathbf{x},t), \qquad \omega \in \Omega$$

time-dependent basis varies orthogonally to V_S



Orthogonality

$$\frac{d\mathbf{V}_S}{dt} \perp \mathbf{V}_S \Leftrightarrow \left\langle \frac{\partial \mathbf{u}_i(\bullet,t)}{\partial t}, \mathbf{u}_j(\bullet,t) \right\rangle = 0 \;, \; i = 1,...,s \;, \quad j = 1,...,s.$$

Sapsis and Lermusiaux, Physica D, (2009)

Dynamically Orthogonal Field Equations, II

$$\mathbf{u}\left(\mathbf{x},t;\omega\right)=\bar{\mathbf{u}}\left(\mathbf{x},t\right)+\sum_{i=1}^{s}Y_i\left(t;\omega\right)\mathbf{u}_i\left(\mathbf{x},t\right),\qquad \omega\in\Omega$$

$$\frac{dY_i\left(t;\omega\right)}{dt}=\left\langle \mathcal{L}\left[\mathbf{u}\left(\bullet,t;\omega\right);\omega\right]-E^{\omega}\left[\mathcal{L}\left[\mathbf{u}\left(\bullet,t;\omega\right);\omega\right]\right],\mathbf{u}_i\left(\bullet,t\right)\right\rangle,$$

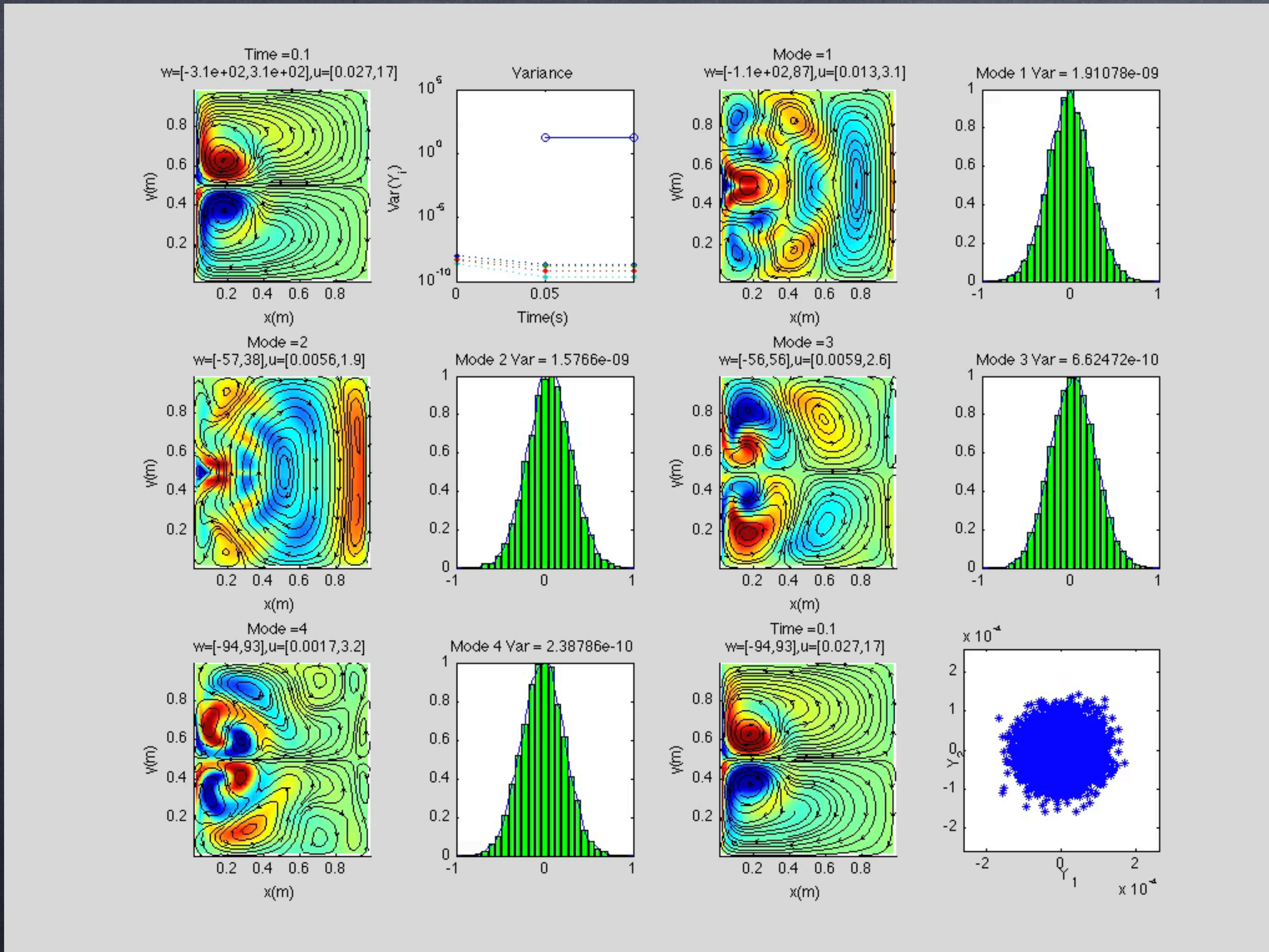
$$\frac{\partial\bar{\mathbf{u}}\left(\mathbf{x},t\right)}{\partial t}=E^{\omega}\left[\mathcal{L}\left[\mathbf{u}\left(\mathbf{x},t;\omega\right);\omega\right]\right],$$

$$\frac{\partial\mathbf{u}_i\left(\mathbf{x},t\right)}{\partial t}=\mathbf{\Pi}_{\mathbf{V}_S^{\perp}}\left[E^{\omega}\left[\mathcal{L}\left[\mathbf{u}\left(\mathbf{x},t;\omega\right);\omega\right]Y_j\left(t;\omega\right)\right]\right]\mathbf{C}^{-1}_{Y_i(t)Y_j(t)}$$

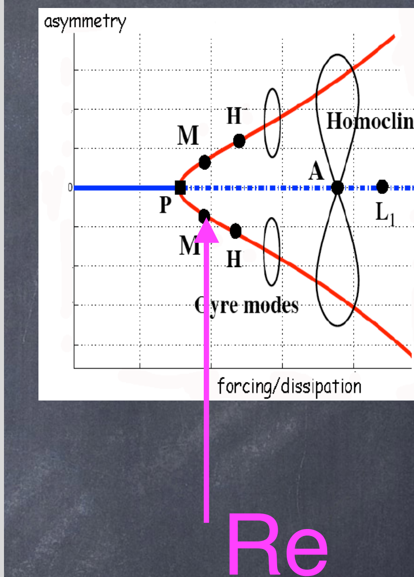
s + 1 deterministic PDEs

system of s SDEs (solve with ensemble size n)

Typical results: double-gyre flow



$s = 4$



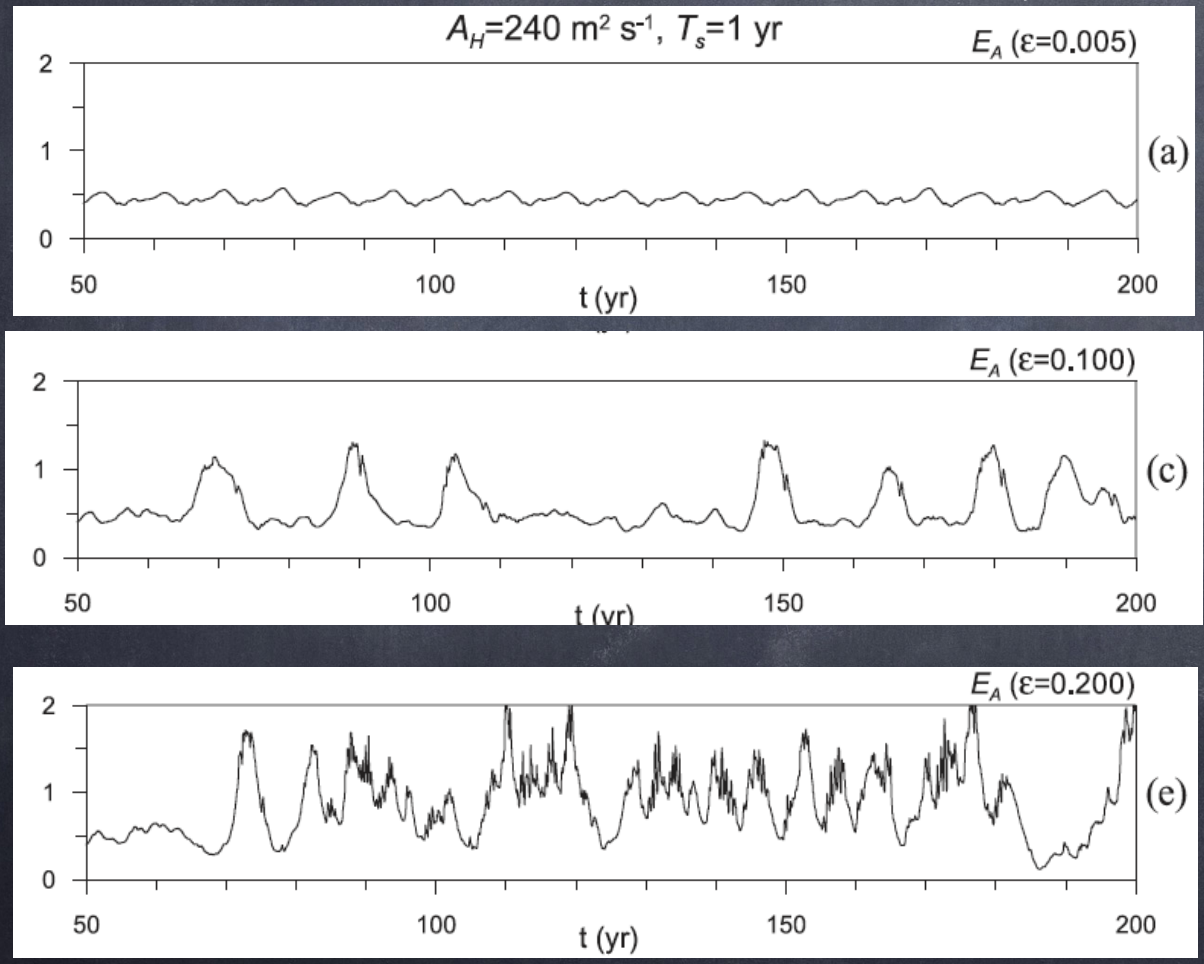
Red noise wind stress can easily excite variability in stable deterministic systems

Sapsis and Dijkstra, JPO, (2013)

Minimal model:
Effect of additive noise in the wind stress

$$\tau(x, y, t) = (1 + \epsilon\zeta(t))\tau_0(x, y)$$

red noise, decorrelation time 1 yr

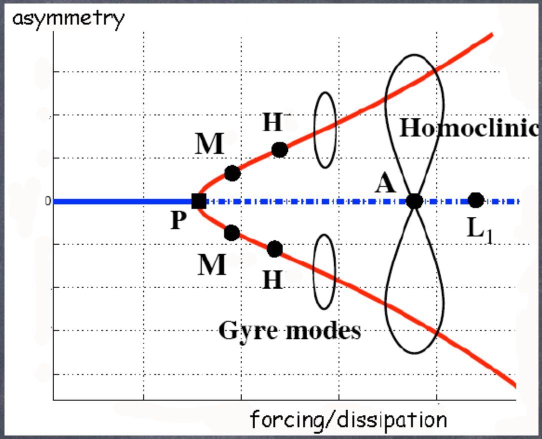


no excitation
for white noise!

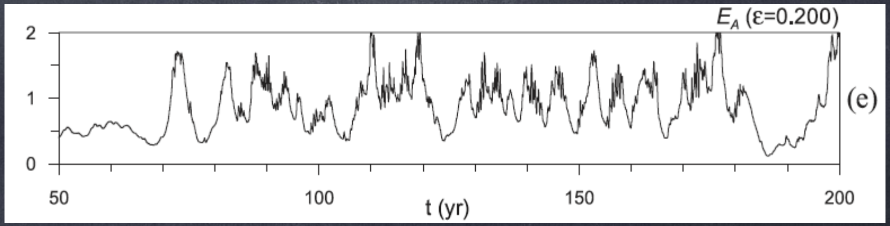
Pierini, JPO, (2009)

Summary Example 1: bifurcation theory Ocean western boundary current variability

Internal variability of the barotropic wind-driven ocean circulation arises through internal (gyre) modes of variability and/or global bifurcations



Temporal correlations in the noise of the wind-stress forcing excite low-frequency internal variability



… but there are more relevant processes

Baroclinic instability and the effect of (meso-scale) eddies

`Turbulent oscillator'

Berloff et al., JPO, (2007)

The effect of sea surface temperature anomalies on wind-stress anomalies

Wind stress anomalies over the Pacific are well correlated with Kuroshio induced SS (and temperature) variations

Qiu et al., J Clim, (2014)

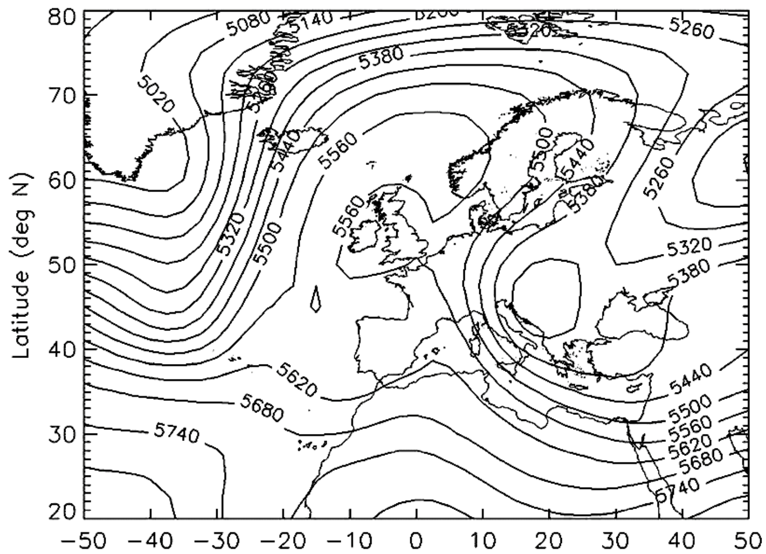
The effect of an external time-dependent wind stress (and Rossby waves)

An external (decadal varying) wind forcing modulates the internal variabil

Pierini, J Clim, (2014)

These have not been included yet into the dynamical systems picture

Example 2:
Midlatitude Atmospheric Flow Transitions



Nearly steady anticyclone deviates westerly jet meridionally →
Blocking event:

- *Recurrent*: ∼ 27 events a year (N. hemisphere)
- *Persistent*: ∼ 5 to 30 days

Problem:
Develop an early warning indicator for a transition from zonal to a blocked state

Tantet et al., Chaos, (2015)

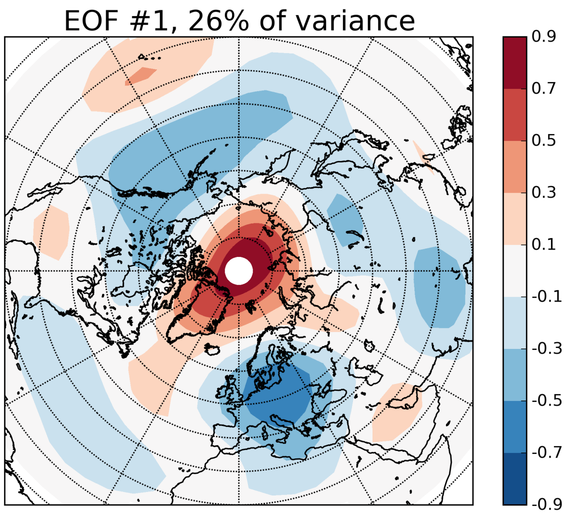
Minimal model

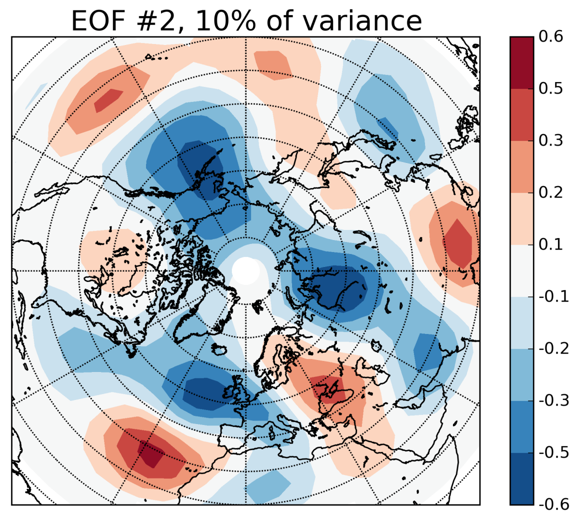
Barotropic T21 model [Selten, 1995]

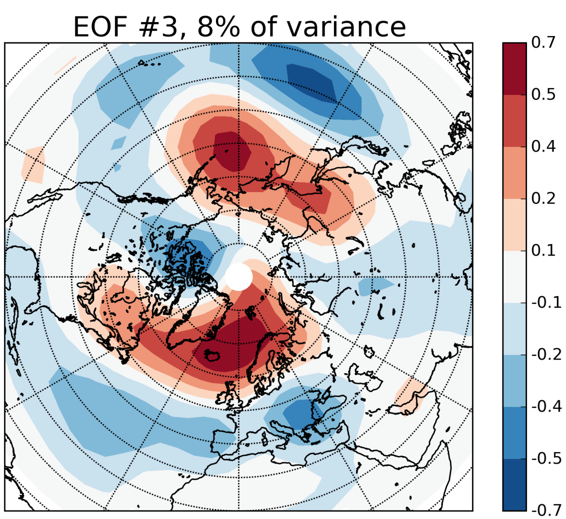
$$\frac{\partial \nabla^2 \psi}{\partial t} = -\mathcal{J}(\psi, \nabla^2 \psi + f + h) - k_1 \nabla^2 \psi + k_2 \nabla^8 \psi + \nabla^2 \psi^*$$

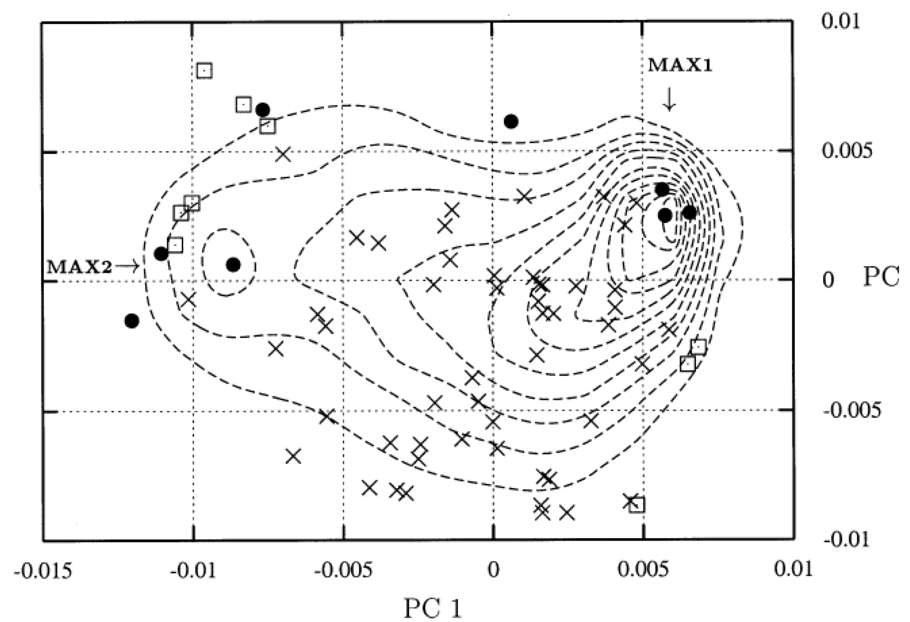
- Realistic topography and winter forcing of N. hemisphere,
- Ekman + scale-selective damping,
- $500,000$ days long simulation.

Variability





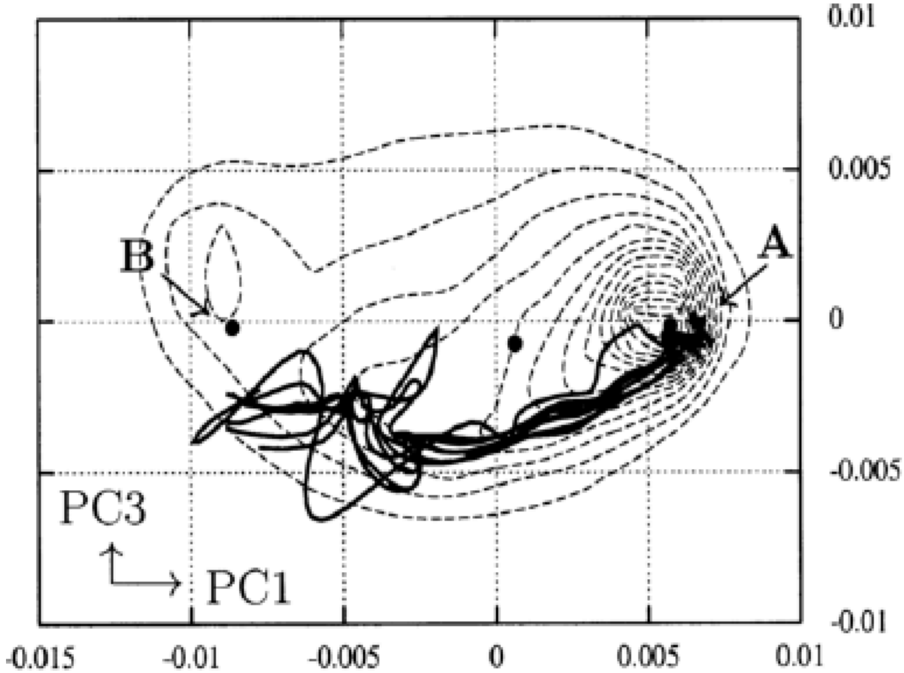




Crommelin, JAS, (2003)

Many unstable fixed points

Reduced dynamics



Recurrent meta-stable regimes

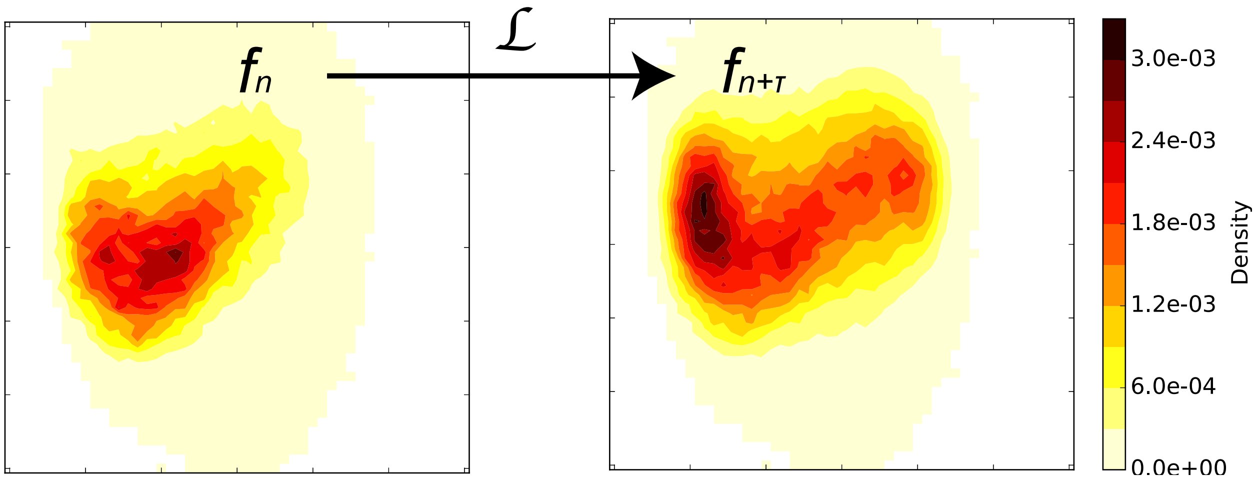
Select an truncated basis of Empirical Orthogonal Functions:

- with largest *decorrelation times*:
  $\rightarrow \tau(\mathrm{pc}_1) = 18d, \quad \tau(\mathrm{pc}_2) = 15d, \quad \tau(\mathrm{pc}_3) = 10d$
- Keep *meta-stability* and *preferred transition paths*:
  $\rightarrow$ reduced phase-space $X = (\mathrm{EOF}_1, \mathrm{EOF}_3)$
  [Crommelin, 2003]

Transfer operator of reduced dynamics

$$f_{n+\tau}(y) = \mathcal{L}_\tau f_n(y) + \text{memory}$$
$$= \int_{\mathbb{R}^d} f_n(x) f_{x_n, x_{n+\tau}}(x, y)\, dx + \text{memory}$$

[Givon et al., 2004]



- How to estimate $\mathcal{L}_\tau$?
- Can we satisfy Markov approximation $\mathcal{L}_{t+s} = \mathcal{L}_t \mathcal{L}_s$?

Estimation of the transfer operator

Galerkin approximation of $f_{x_n, x_{n+\tau}}$ by a transition matrix P_τ [Froyland, 1998, Dellnitz and Junge, 1999, Chekroun et al., 2014]:

- Define a grid of disjoint boxes $\{B_i\}_{1 \leq i \leq m}$ supporting the attractor
- Estimate the transition probabilities $P_{\tau,ij} = \mathbb{P}(x_{n+\tau} \in B_j | x_n \in B_i)$ approximating $\mathcal{L}_\tau$.

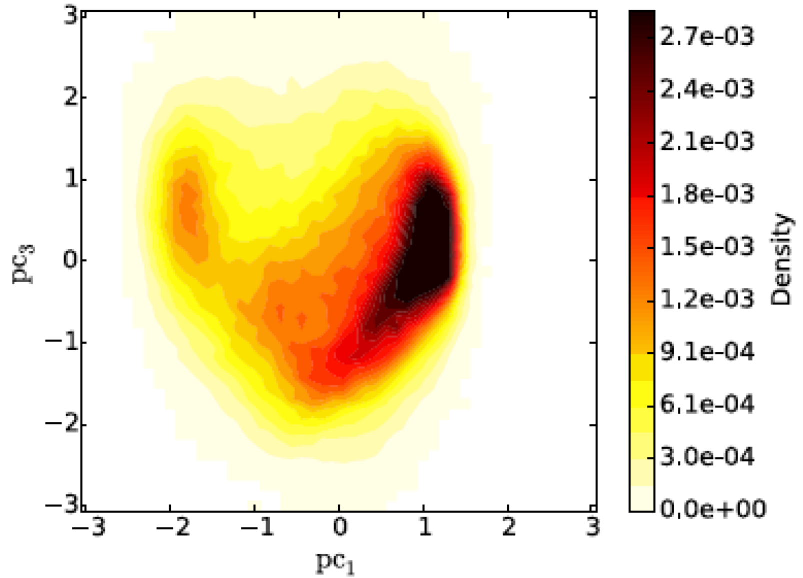
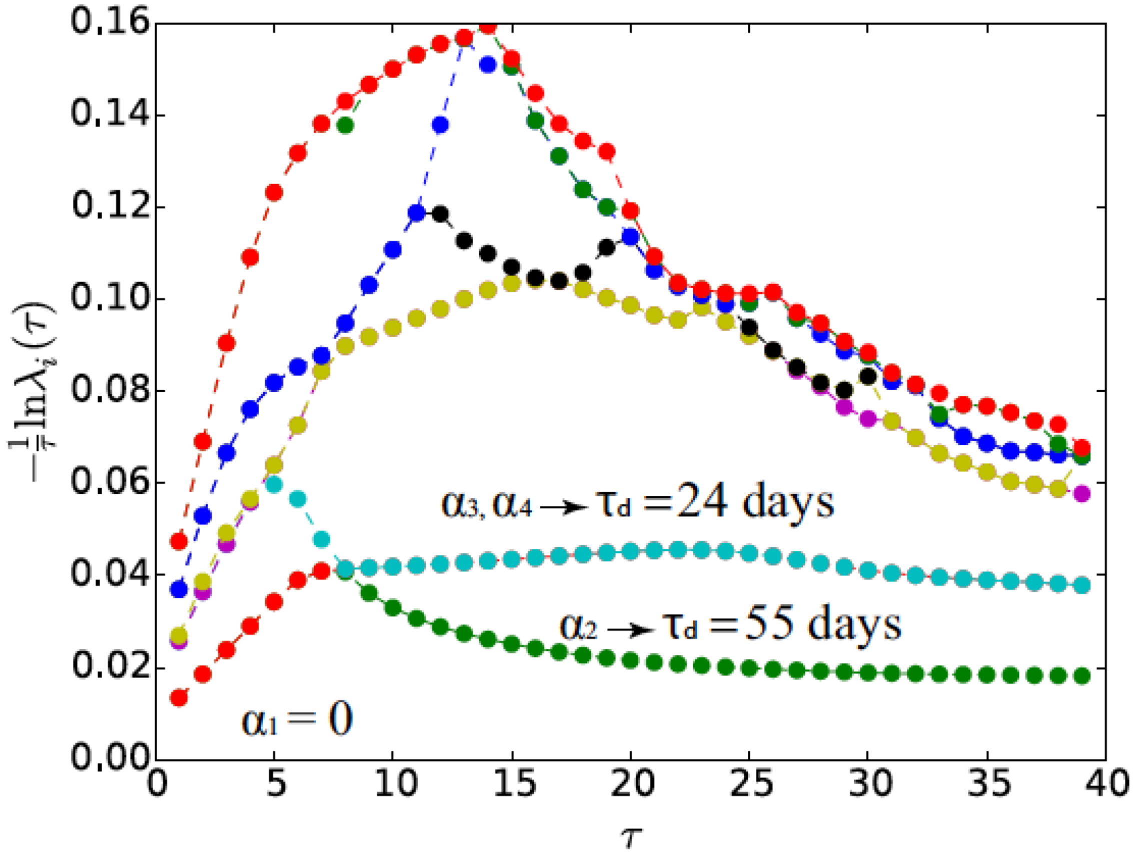


Figure: 1st *left-eigenvector → stationary density estimate.*

Spectral properties

Eigenfunctions of $\mathcal{L}_\tau$ $\rightarrow$ left eigenvectors of $\hat{P}_\tau$.

Decorrelation rates: $\alpha_k = -\frac{1}{\tau}\ln\lambda_k \rightarrow$ Constant if Markovian!



- Spectral gap for $\tau > 5$ days $\rightarrow$ time-scale separation,
- Dominant rates constant w.r.t τ for $\tau > 8$ days $\rightarrow$ Markovian.

Almost invariant sets: results

$$\mathbb{P}(x_{t+\tau} \in A | x_t \in A) \approx 1.$$

Optimal almost invariants: optimal Markov chain reduction
(Deng et al.,IEEE Autom. Control, (2011)).

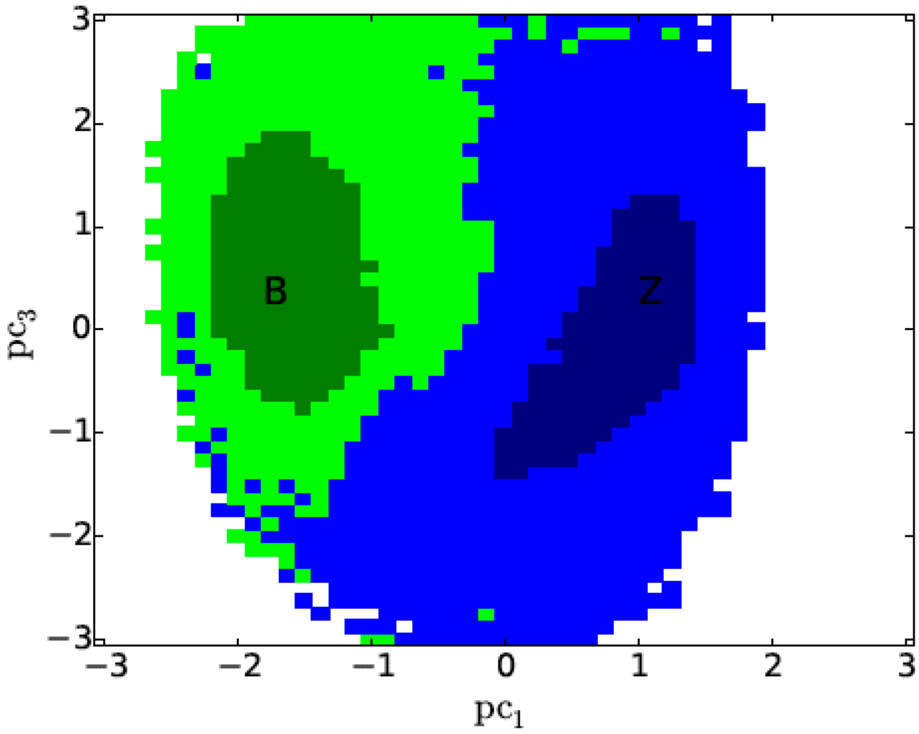
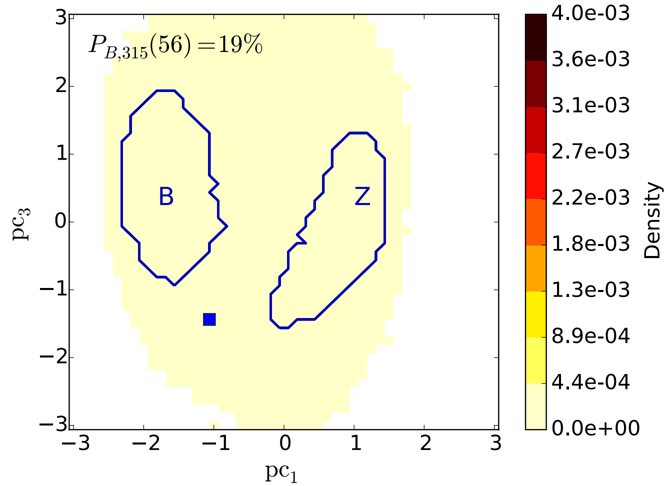
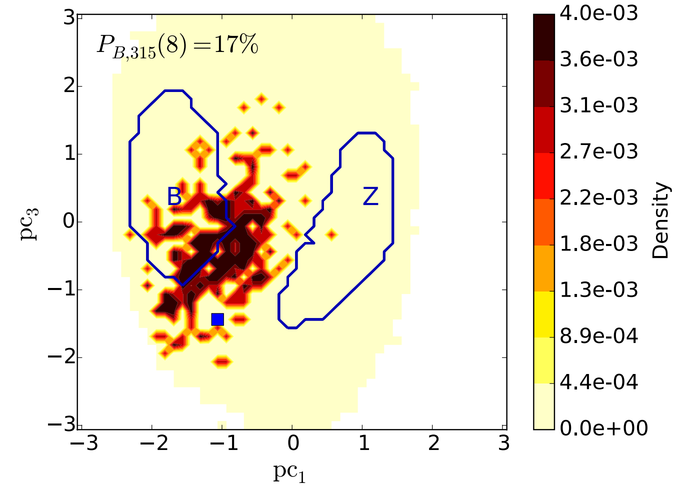


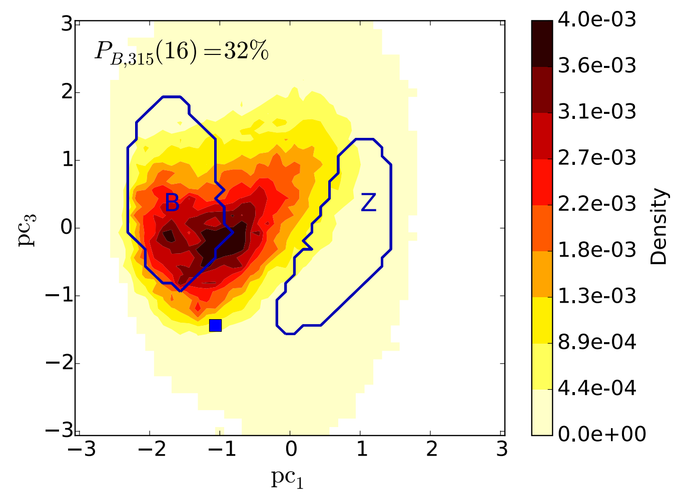
Figure: *Two almost-invariants of* P_τ *for* $\tau = 8$, *with density* 0.27 *(light green) and* 0.73 *(light blue) and the restriction of half of their volume, the B(locked) regime (dark green) and the Z(onal) regime (dark blue).*

Early warning indicator?

- Predict the evolution of initial density f_0: $f_{k\tau} = f_0 P_\tau^k$,
- *Alarm* at time $k\tau$ when $\sum_B P_\tau f_0(y) > p_c$.



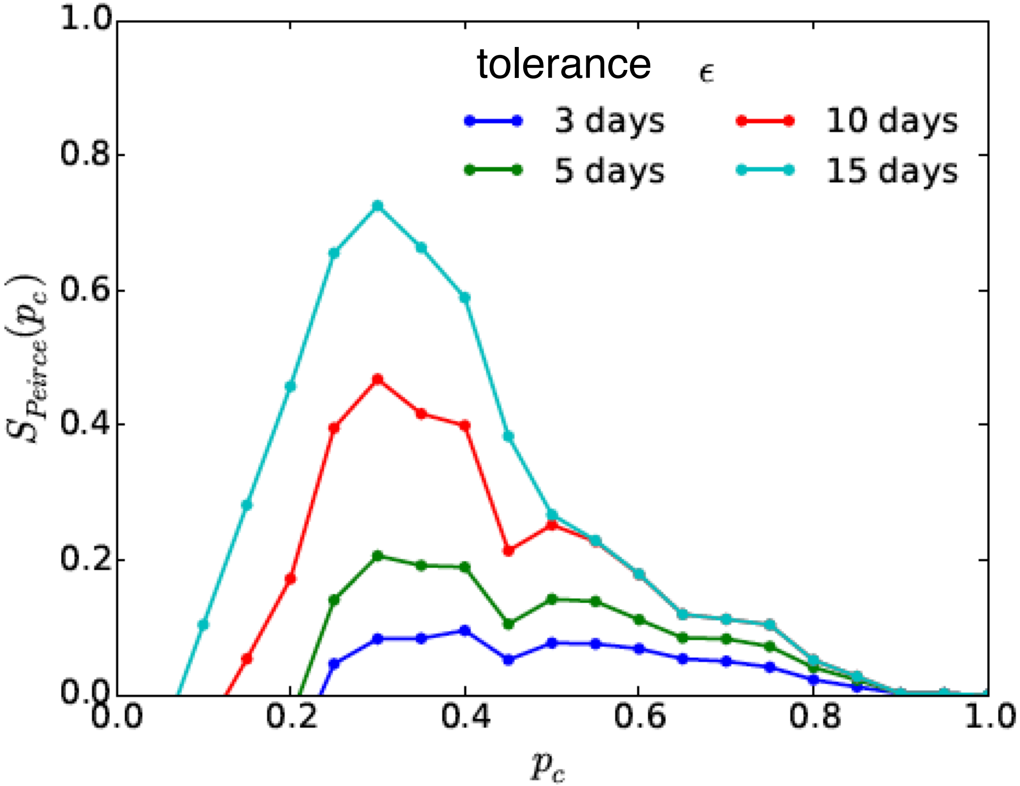




Alarm for p_c = 0.3

Skill of the indicator

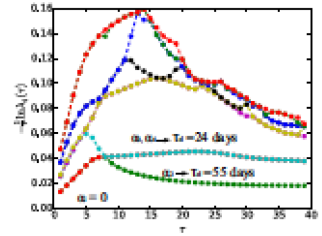
$S_{Peirce} = \textit{hit rate - false alarm rate}$



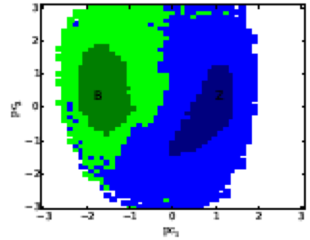
- Best skills for $p_c \sim 0.3$,

Summary example 2

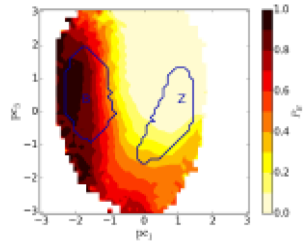
Transfer operator → projected dynamics of high-dimensional system:



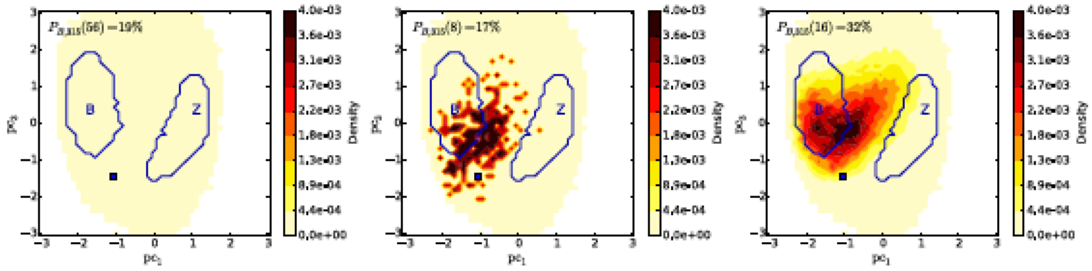
- Time scales and memory effects,



- Objective definition of atmospheric regimes,



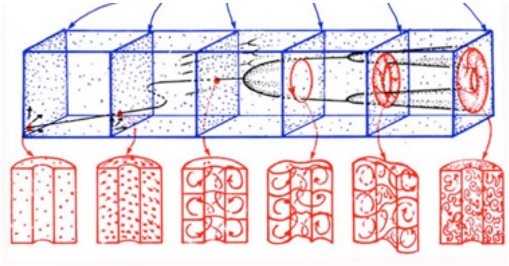
- Preferred path → potential predictability,
- Discrete stochastic prediction.



Overall summary

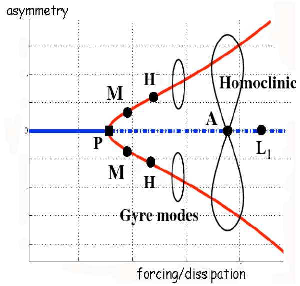
Dynamical systems approach

- Model hierarchy
- Concepts/techniques



I. Behavior involving `low-dimensional' attractors

- Local bifurcation theory
- Global bifurcations



II. Behavior involving `high-dimensional' attractors

- Transfer operator techniques
- Markovian approximations