Direct Statistical Simulation of Geophysical Flows: Taming the Curse of Dimensionality

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"An alternative procedure which does not suffer this disadvantage consists of deriving a new system of equations whose unknowns are the statistics themselves...."

Edward Lorenz, The Nature and Theory of the General Circulation of the Atmosphere (1967)

"Direct Statistical Simulation" (DSS)

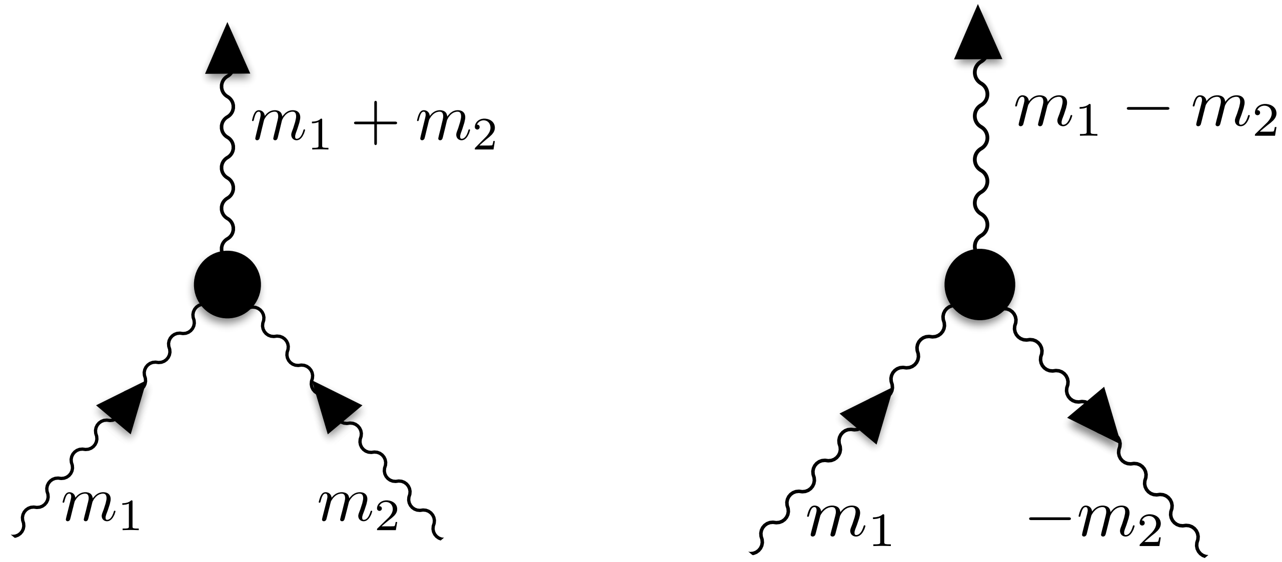
DSS vs. DNS

Low-order statistics are smoother in space than the instantaneous flow.

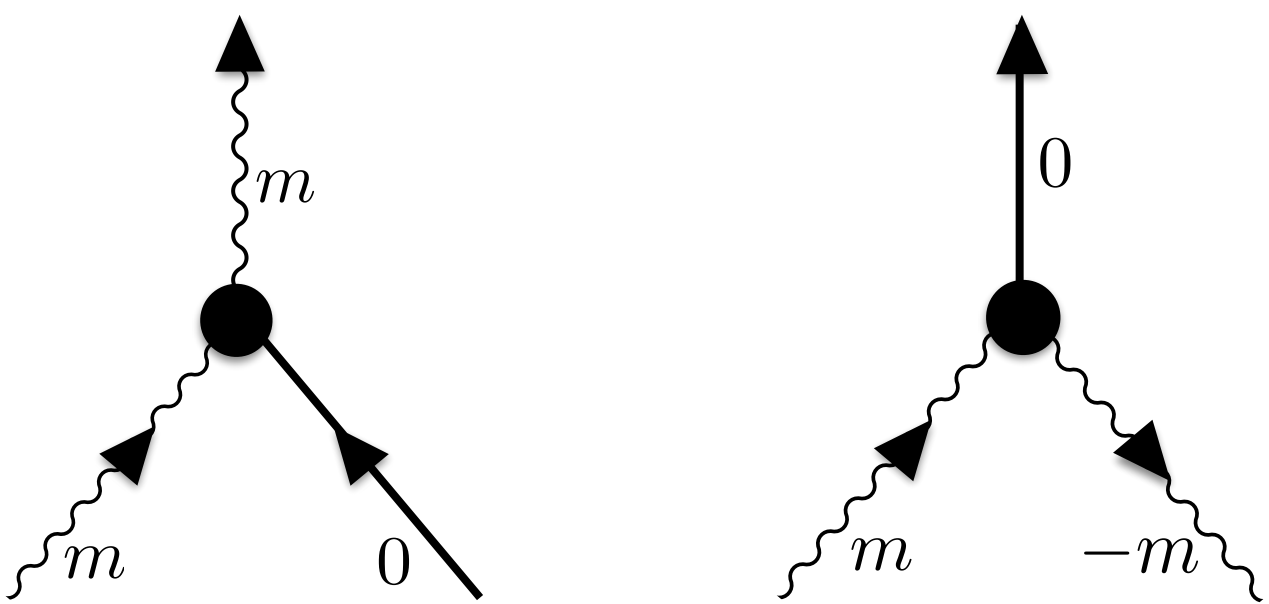
Statistics evolve slowly in time, or not at all, and hence may be described by a fixed point, or at least a slow manifold.

Correlations are *non-local* and highly anisotropic and inhomogeneous. Statistical formulations should respect this.

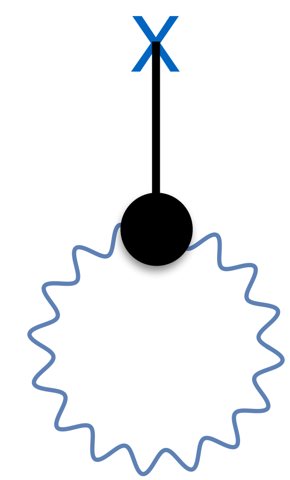
Systems with Zonal Symmetry



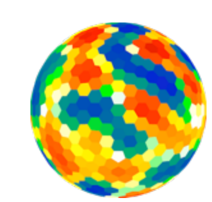
$$q(\phi, \lambda) = \overline{q}(\phi) + q'(\phi, \lambda)$$



QL / S3T / CE2



JBM, W. Qi, and S. M. Tobias, “Direct Statistical Simulation of a Jet” arXiv:1412.0381 (CE2, CE2.5 and CE3). “GCM” on the Apple Mac App Store (2000+ downloads)



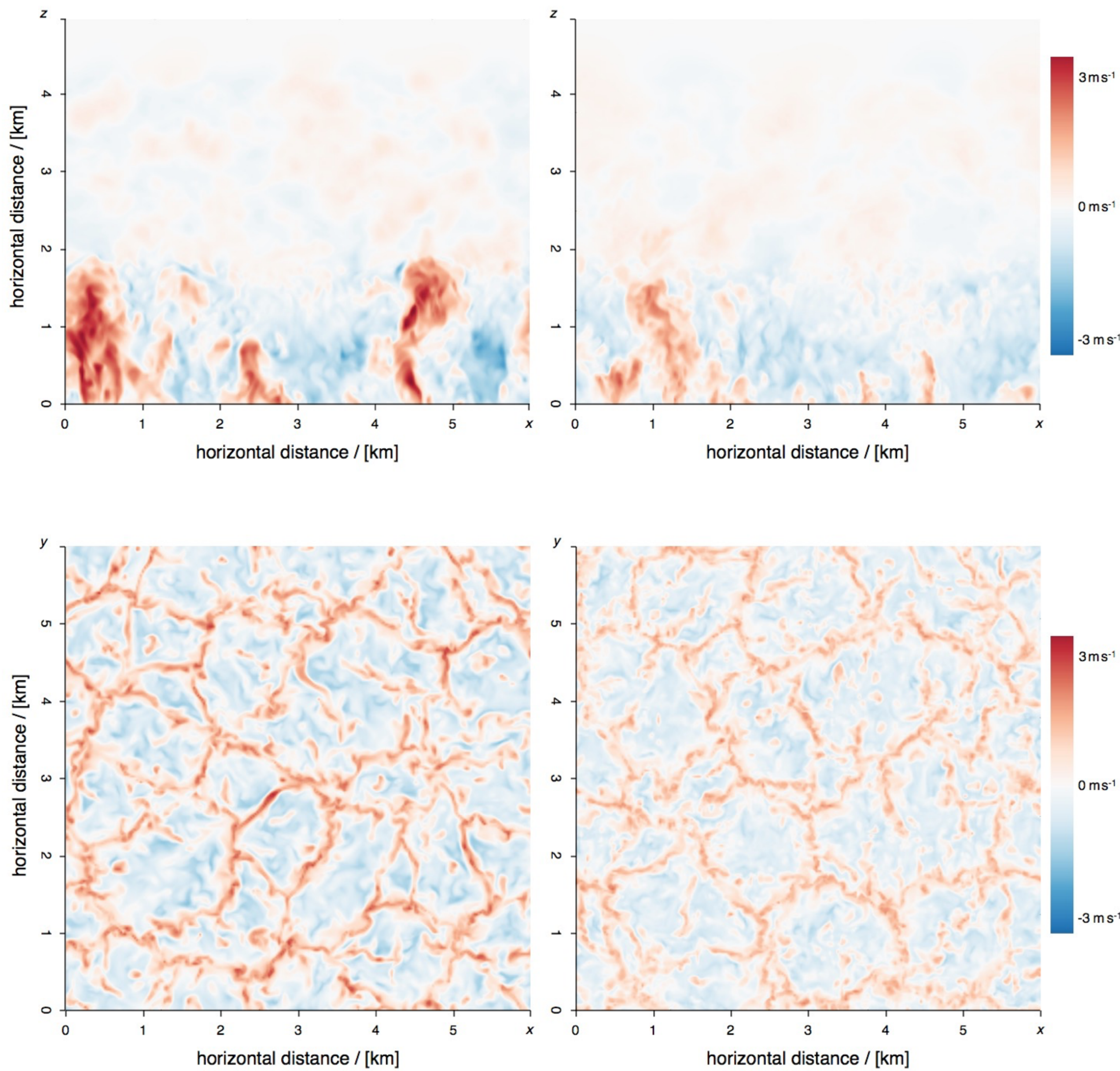
Kill Three Birds With One Stone?

1. Address systems that lack zonal symmetry.

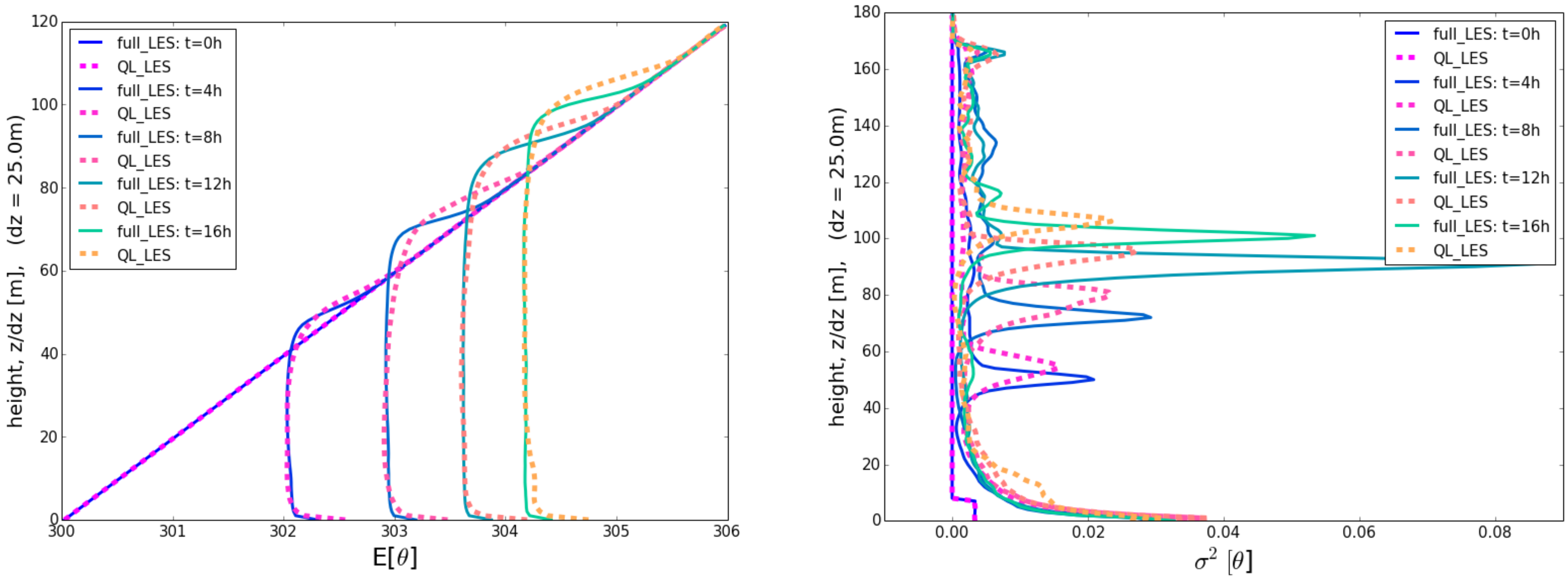
2. Seamlessly integrate sub-grid physics (eg. turbulence, convection, clouds, sub-mesoscale eddies) into GCMs.

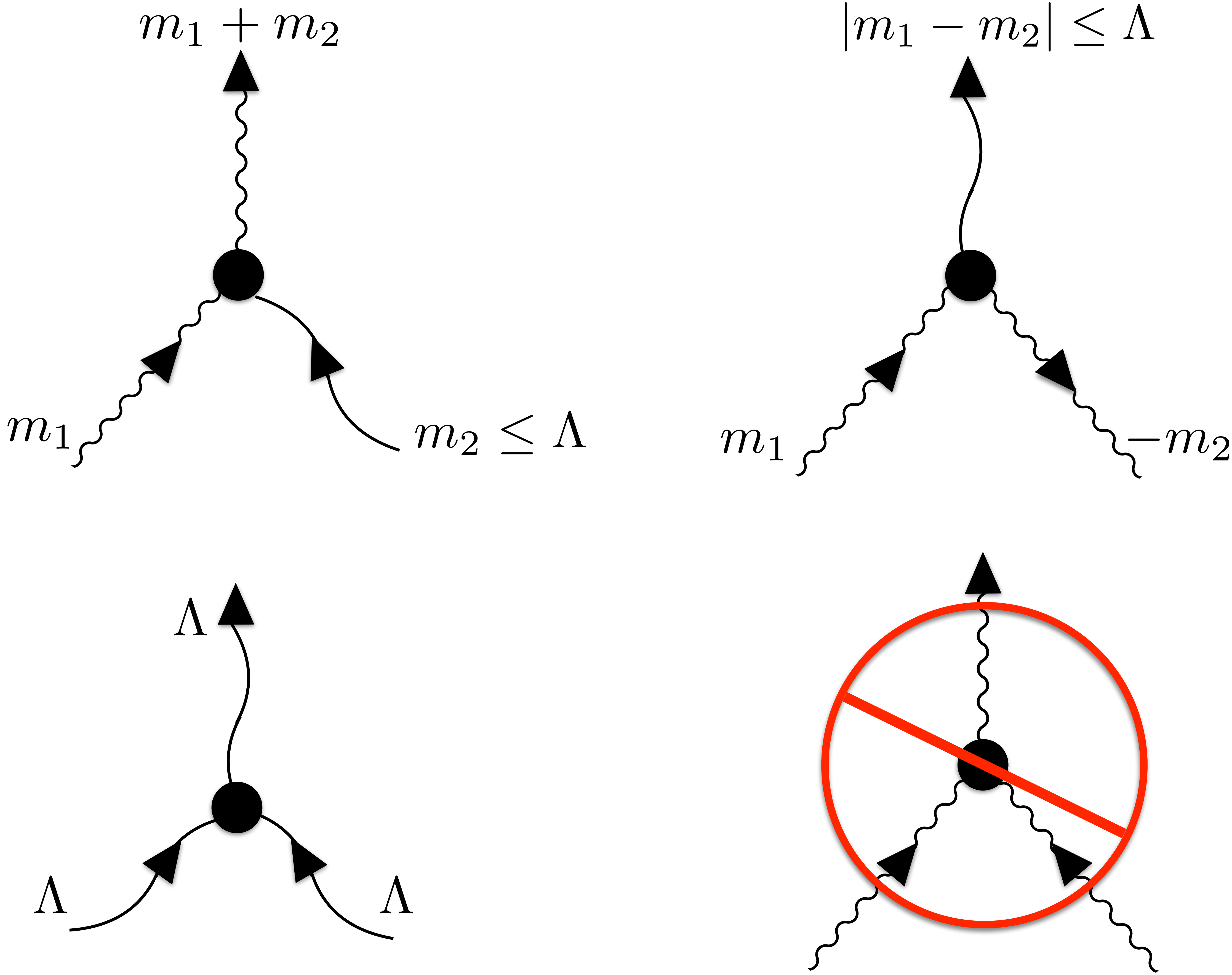
3. Greatly increase speed and resolution.

Large-Eddy-Simulation (LES) of Dry Boundary Layer (fully nonlinear and QL)
See poster “Non-local second order closure scheme for boundary layer turbulence”
by Bettina Meyer and Tapio Schneider



Large-Eddy-Simulation (LES) of Dry Boundary Layer (fully nonlinear and QL)
Bettina Meyer & Tapio Schneider (ETH Zurich)





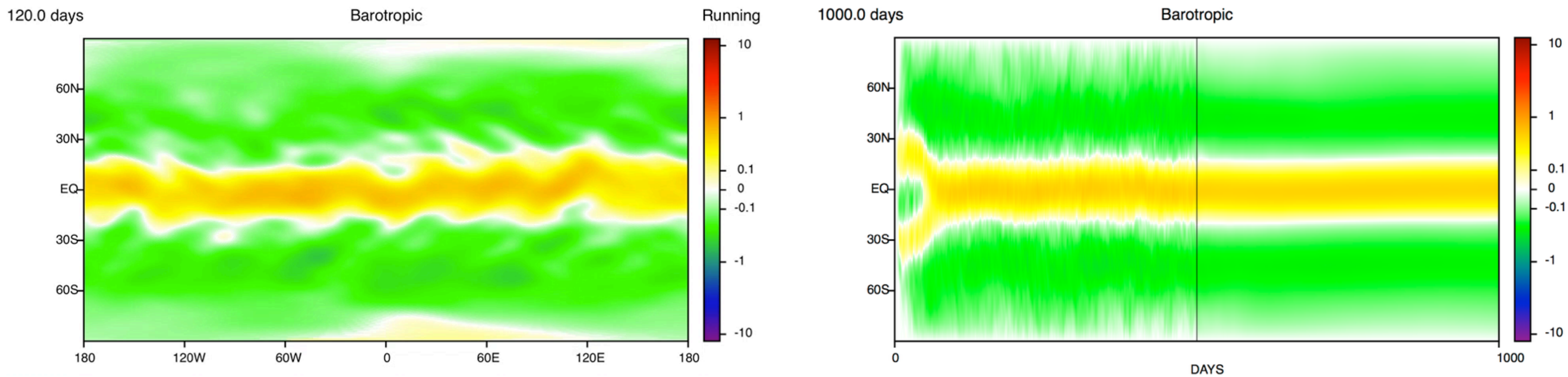
GQL = Generalized Quasilinear DNS

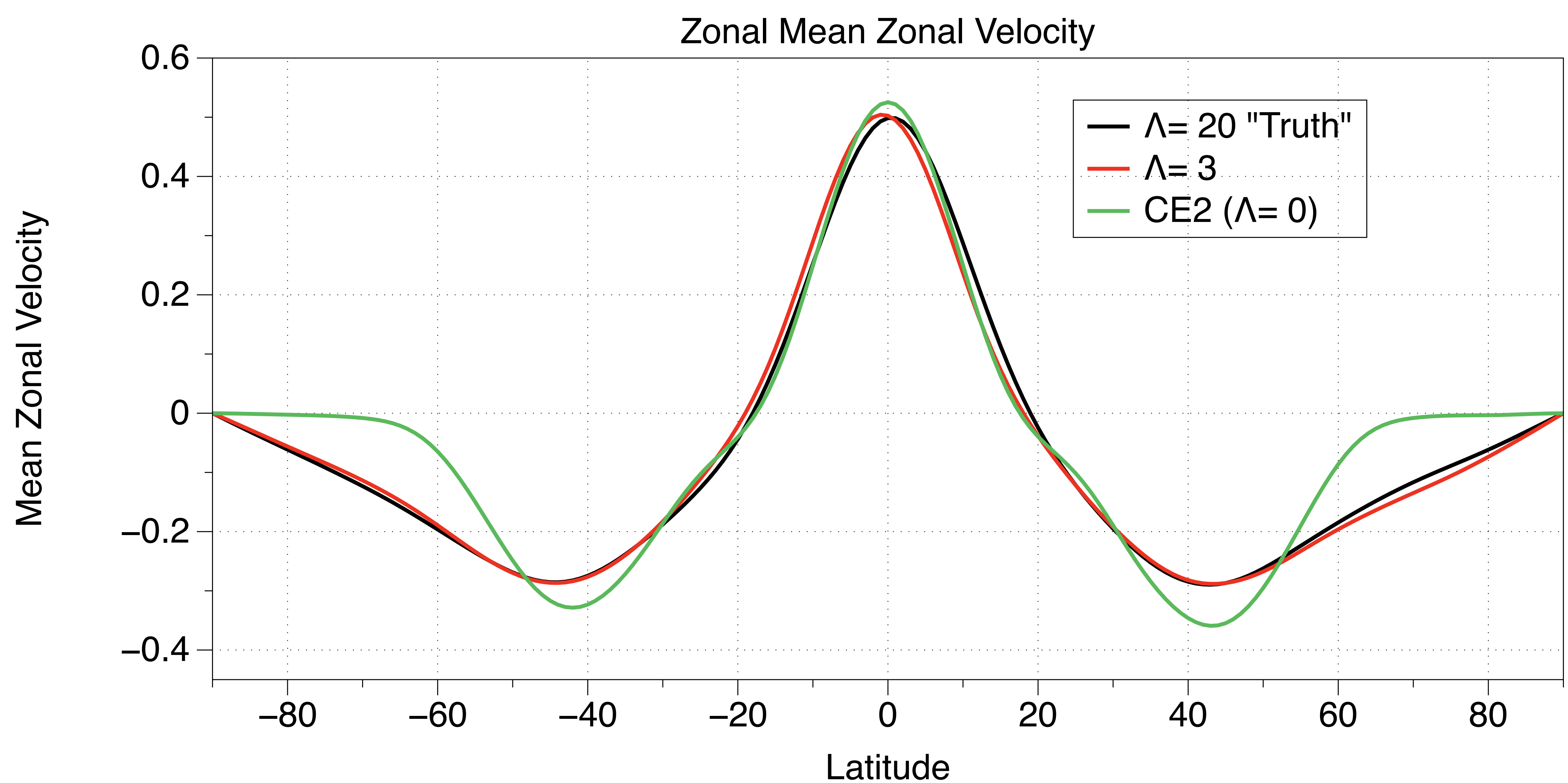
Barotropic Toy Model of Jets

$$\partial_t \zeta + \vec{v} \cdot \vec{\nabla}(\zeta + f) = -\kappa\zeta - \nu_2 \nabla^4 \zeta + \eta$$

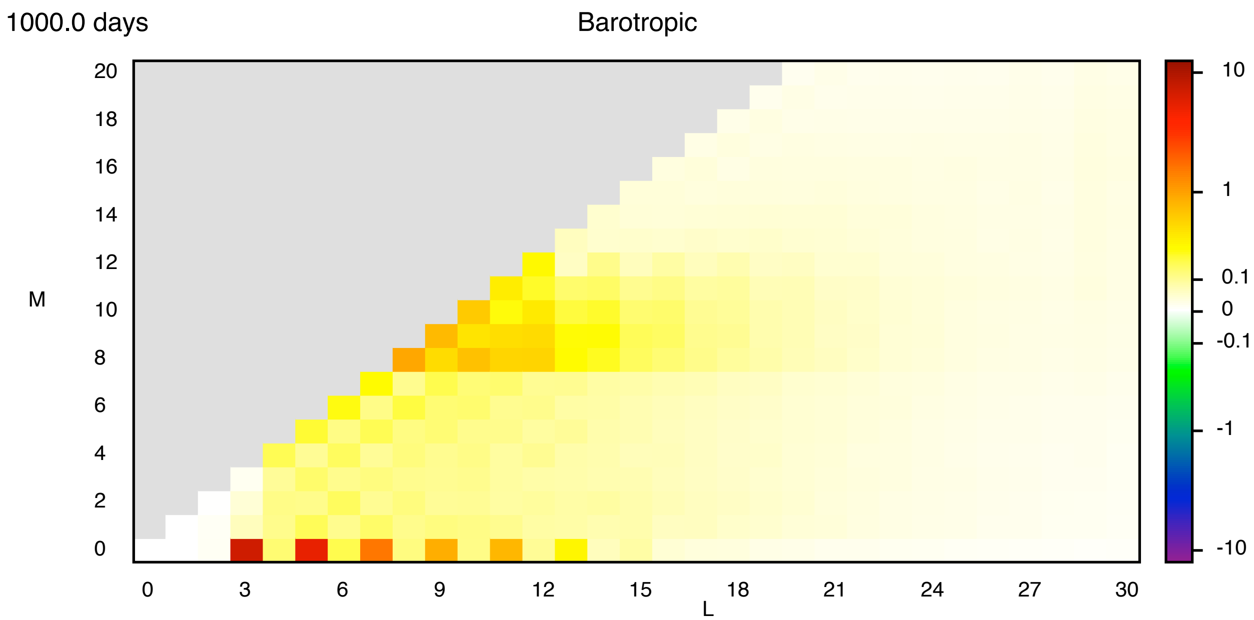
$$\zeta = \hat{r} \cdot \vec{\nabla} \times \vec{v} \qquad\qquad f = 2\Omega \sin(\phi)$$

$$\zeta(\theta, \phi) = \sum_{\ell=0}^{L} \sum_{m=-\ell}^{\ell} \zeta_{\ell,m} Y_\ell^m(\theta, \phi)$$

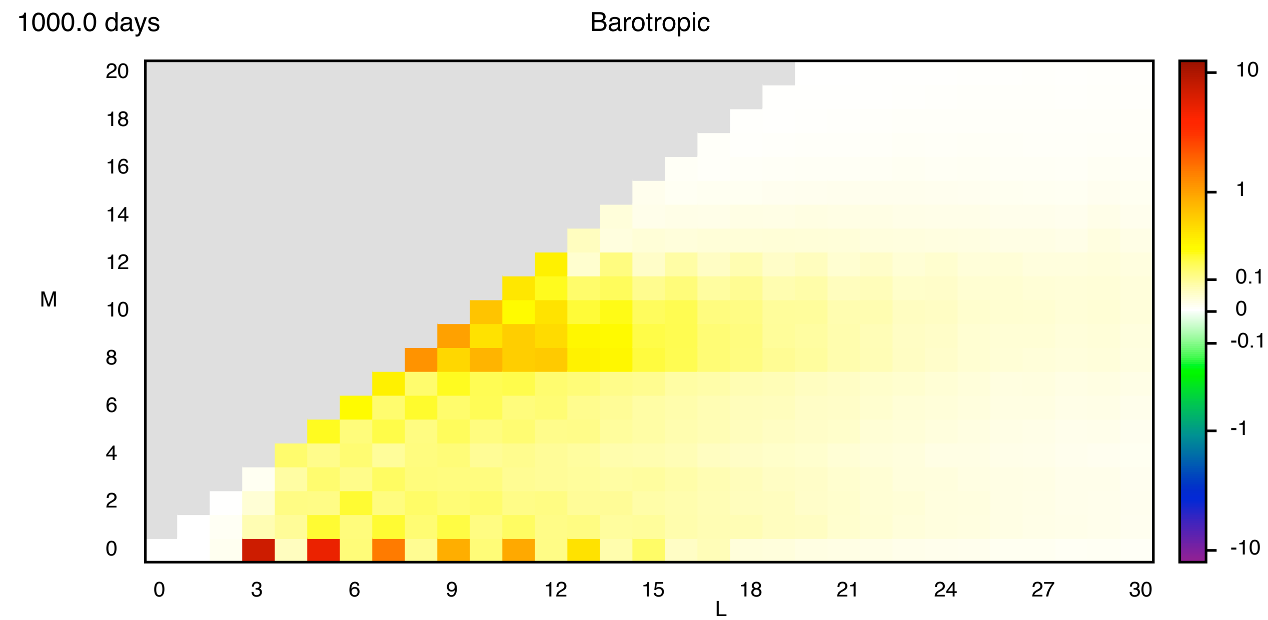

120.0 days
Barotropic
Running
60N
30N
EQ
30S
60S
180
120W
60W
0
60E
120E
180
10
1
0.1
0
-0.1
-1
-10
1000.0 days
Barotropic
DAYS
0
1000


Zonal Mean Zonal Velocity
Λ= 20 "Truth"
Λ= 3
CE2 (Λ= 0)
Mean Zonal Velocity
0.6
0.4
0.2
0
−0.2
−0.4
−80
−60
−40
−20
0
20
40
60
80
Latitude

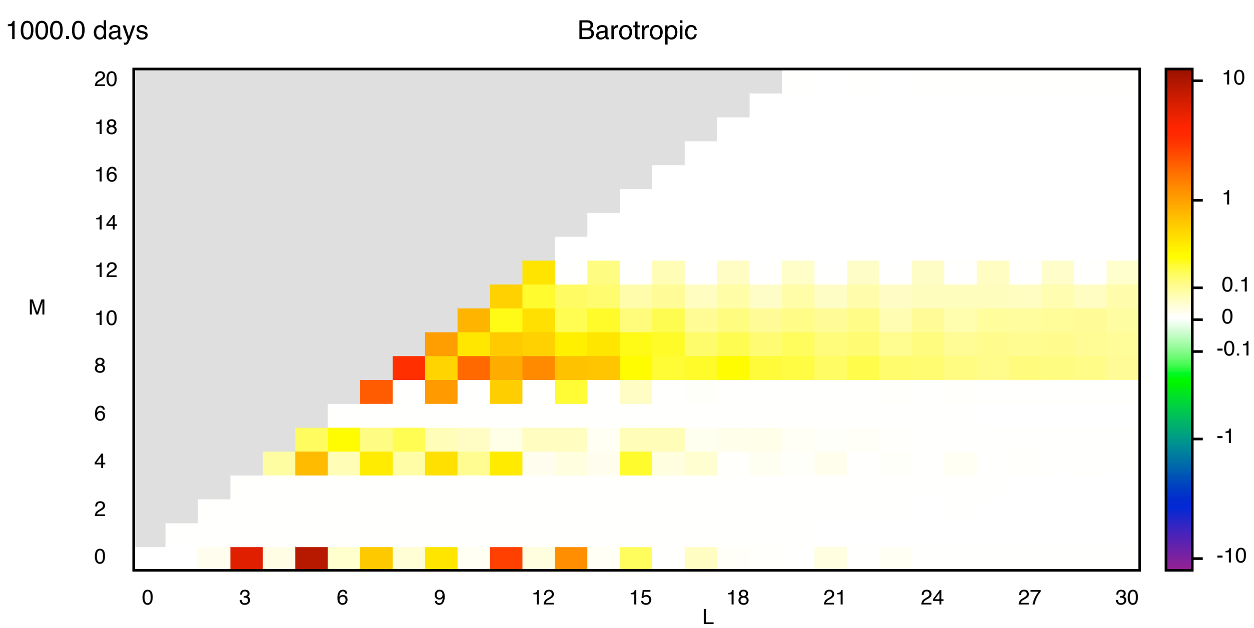
Vorticity Power Spectra



$\Lambda = 20$ “Truth”

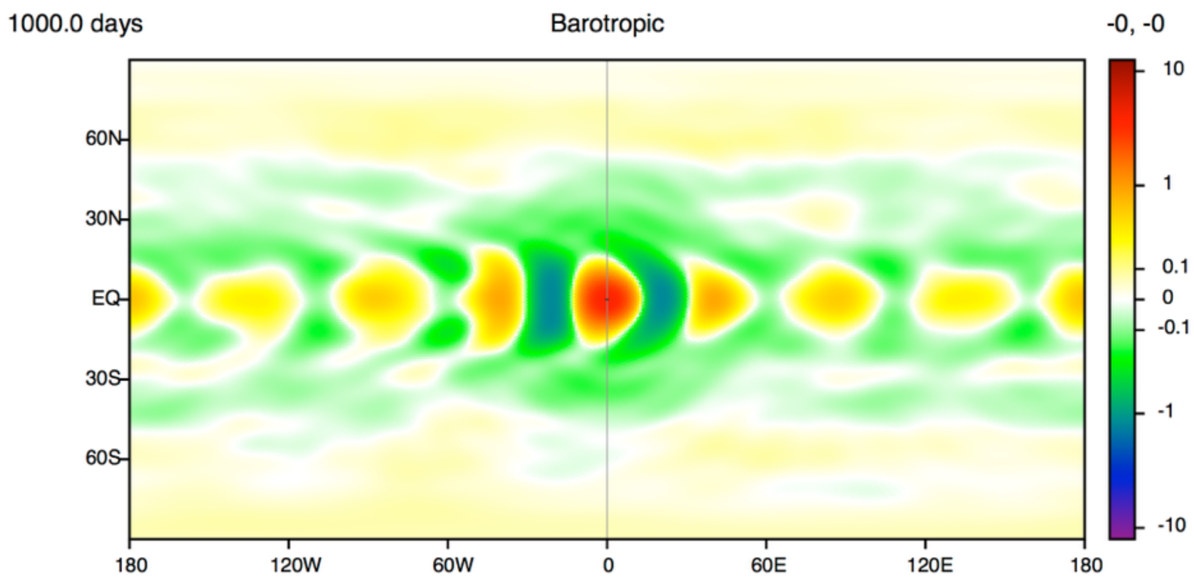


$\Lambda = 3$



$\Lambda = 0$

Two-Point Vorticity Correlations



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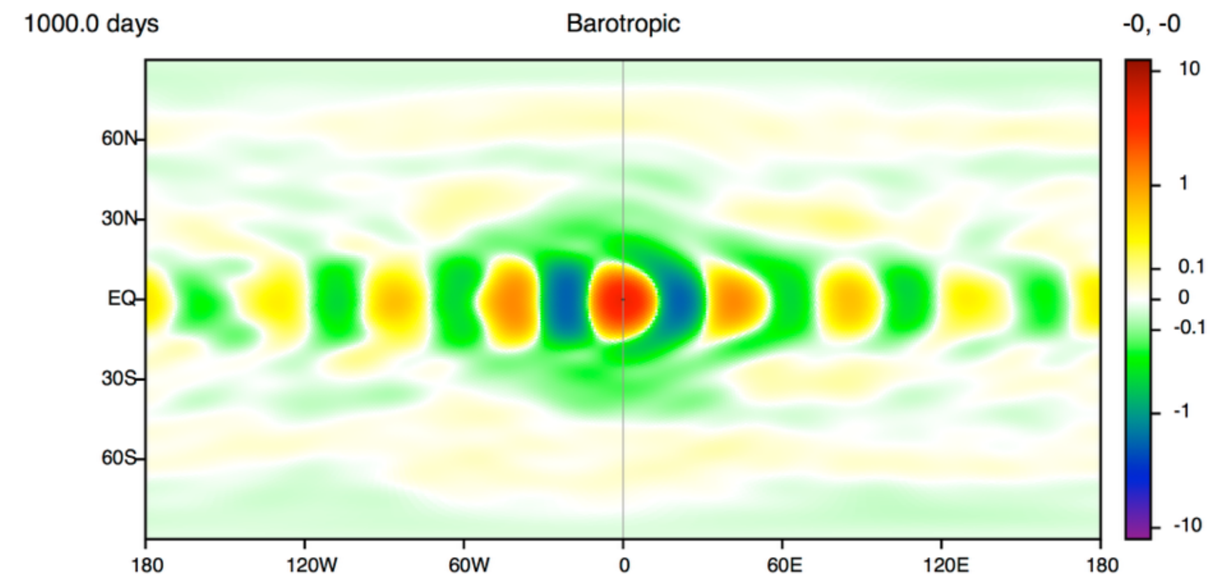
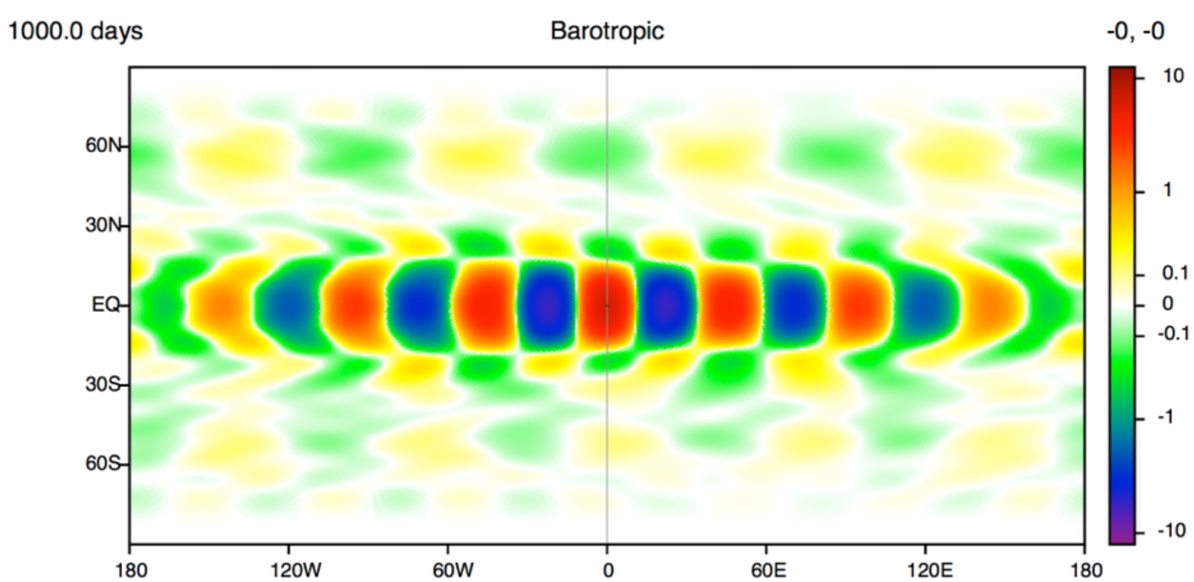


$\Lambda = 3$



$\Lambda = 0$

Generalized 2nd Order Cumulant Expansion (GCE2)

$$\frac{\partial}{\partial t} q = L[q] + Q[q,\ q]$$

$$q = \ell + h$$

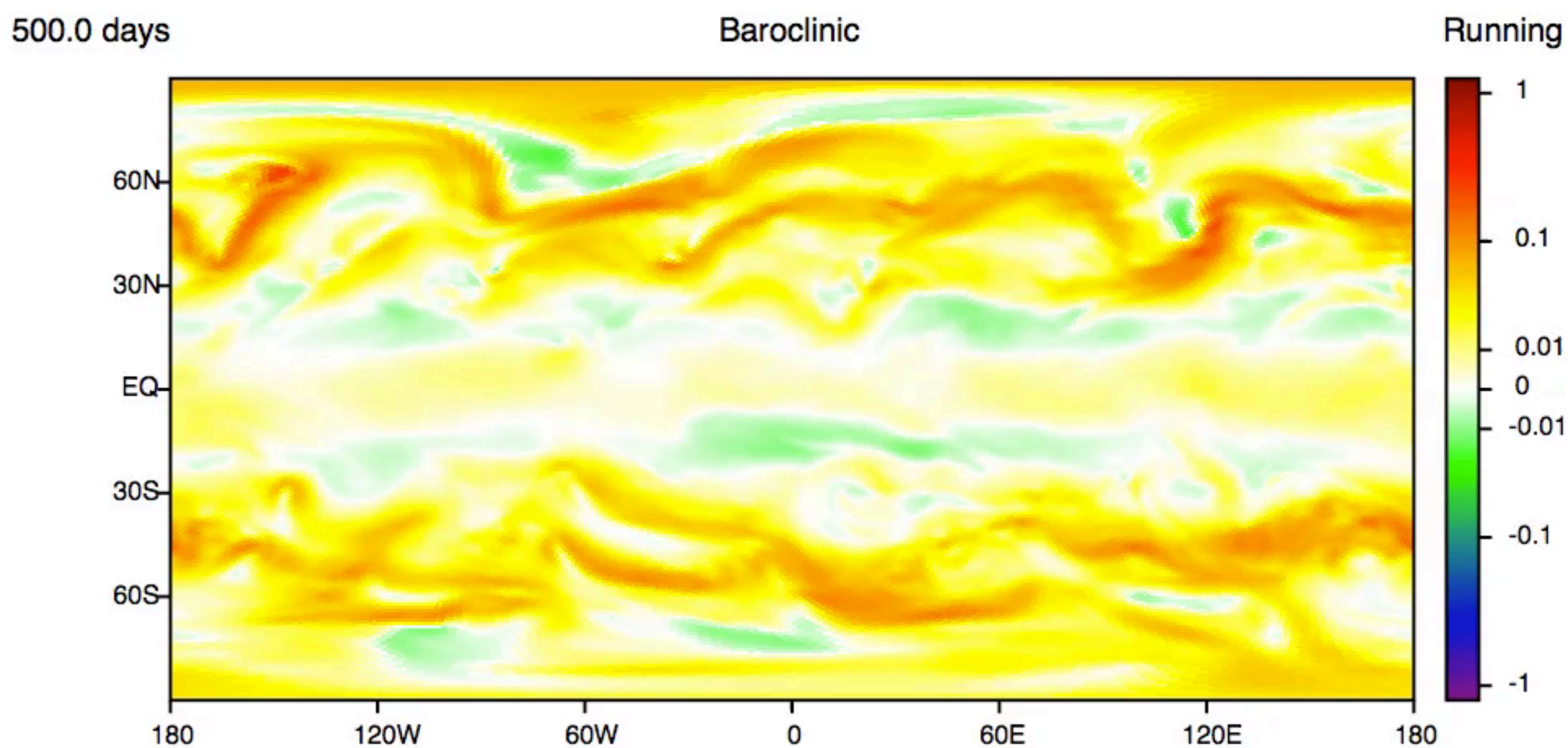
$$\frac{\partial}{\partial t} \ell = Q[\ell,\ \ell] + Q[(h,\ h)] \qquad \frac{\partial}{\partial t} h = Q[\ell,\ h]$$

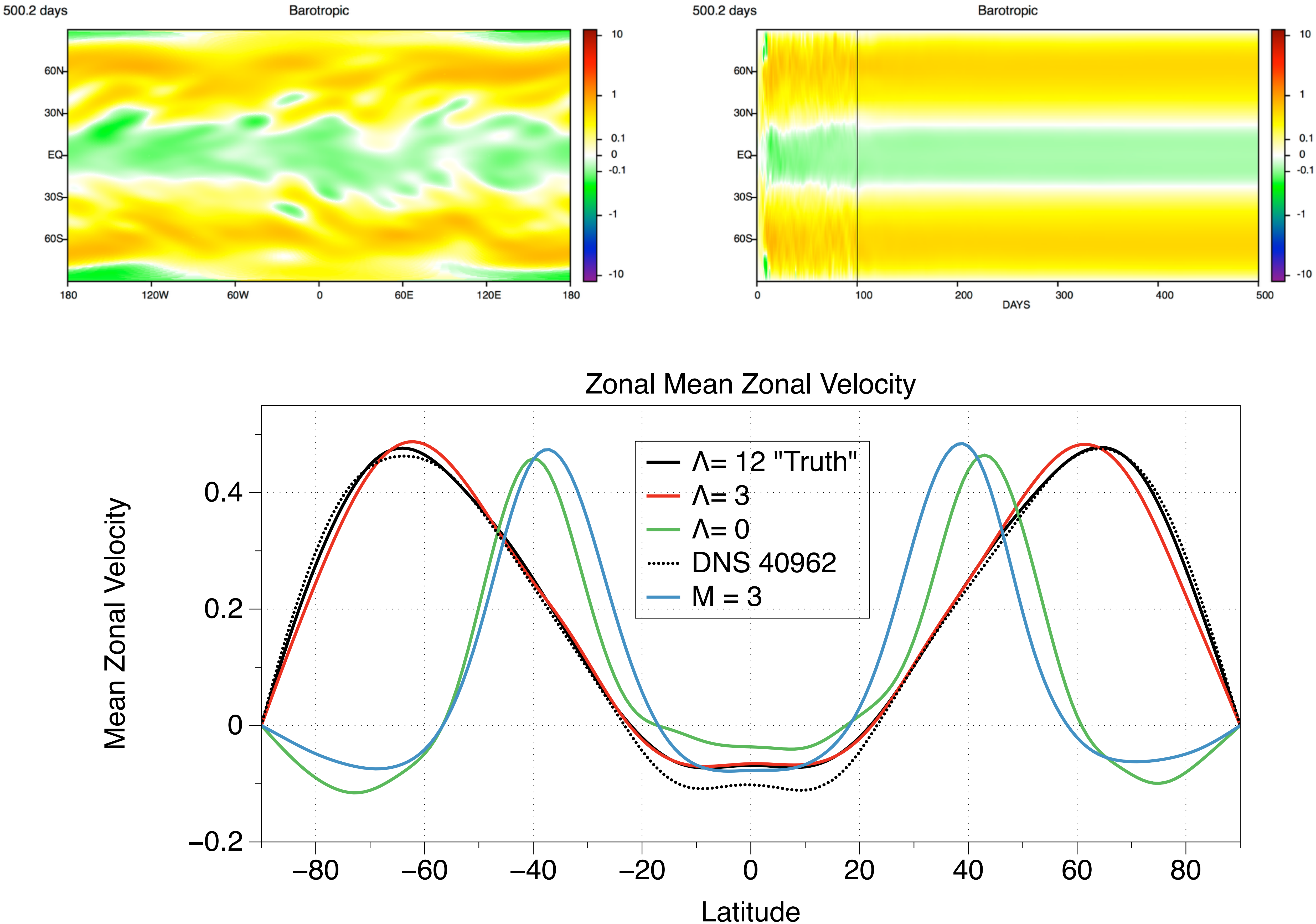
$$\frac{\partial}{\partial t} (h\ h) = 2Q[\ell,\ (h]\ h)$$

Closure

Malecha, Chini, and Julien, J. Comp. Phys. (2013);
Bakas and Ioannou, PRL **110**, 224501 (2013)

Two-layer primitive equations: Relaxation to a prescribed equator-to-pole temperature difference





500.2 days
Barotropic
60N
30N
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1
0.1
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500.2 days
Barotropic
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100
200
300
400
500
DAYS
Zonal Mean Zonal Velocity
Mean Zonal Velocity
Latitude
Λ= 12 "Truth"
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DNS 40962
M = 3
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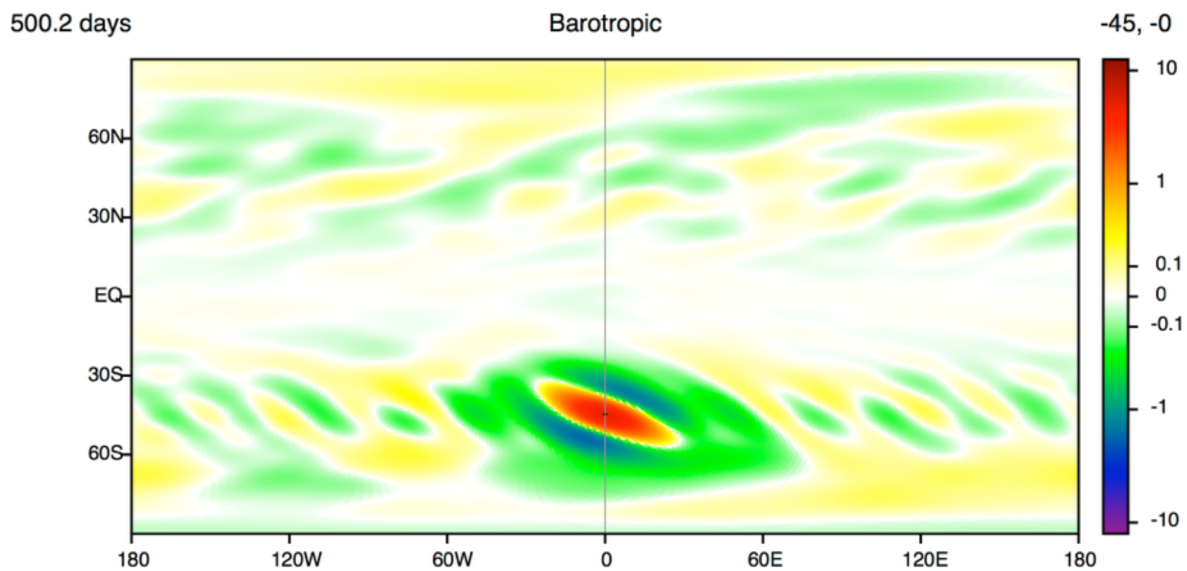
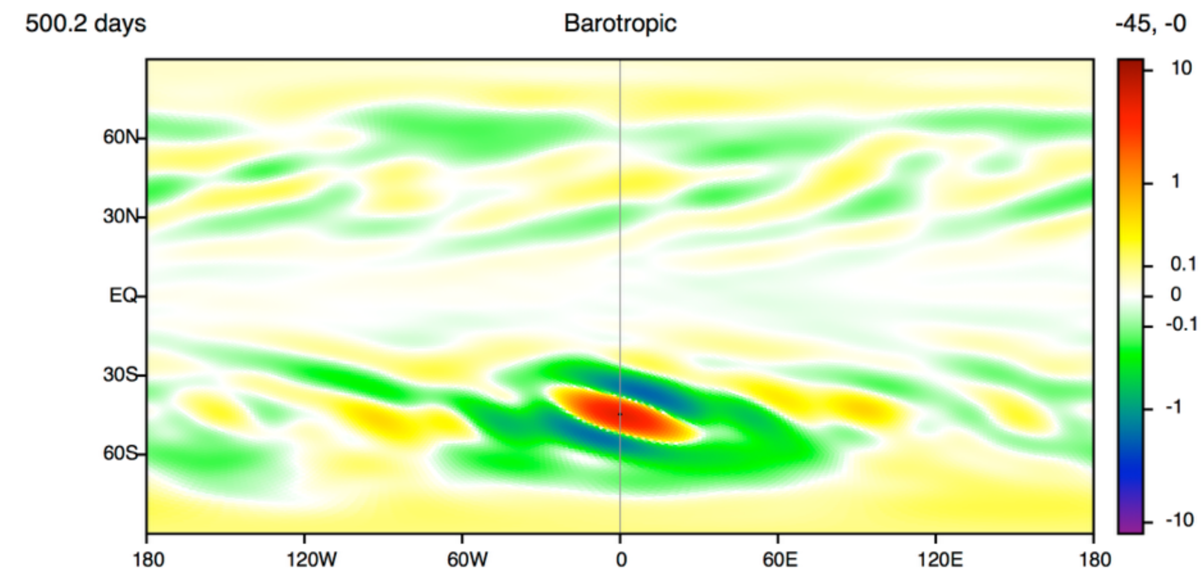
−60
−40
−20
0
20
40
60
80

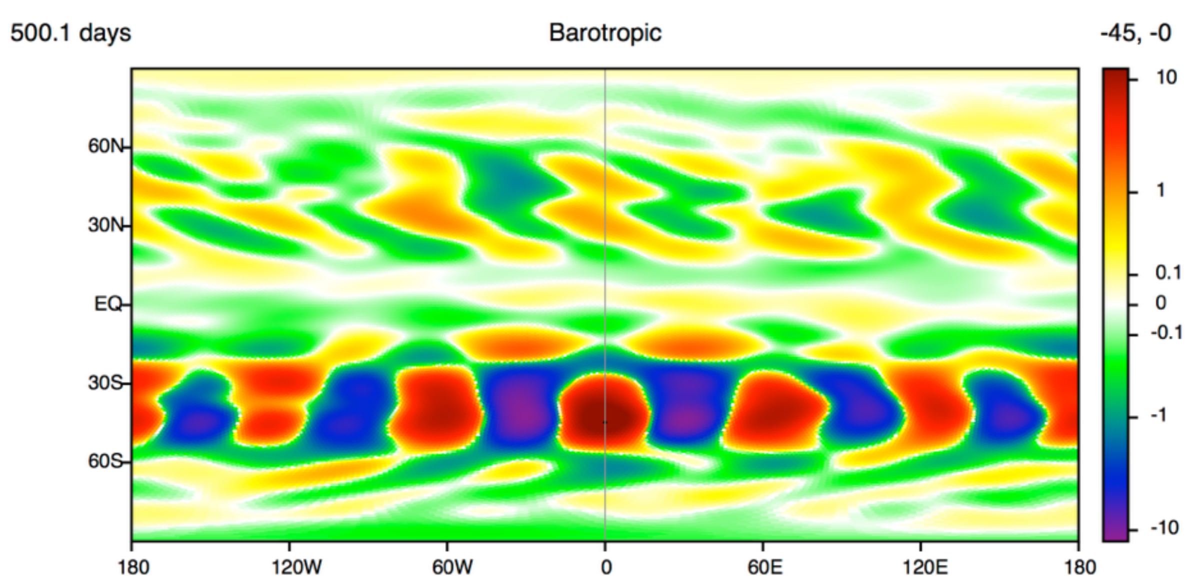
Two-Point Vorticity Correlations



$\Lambda = 12$ “Truth”

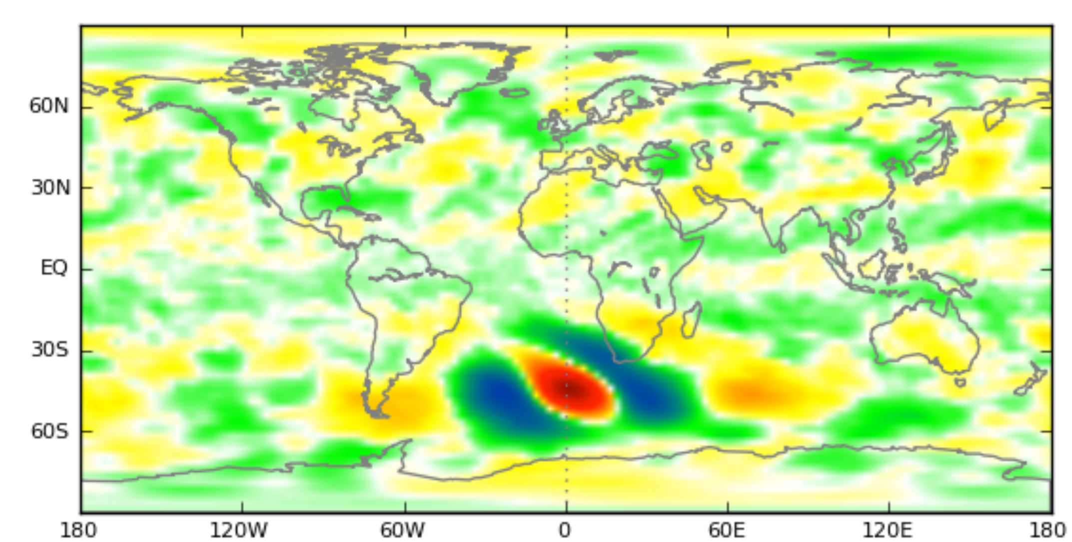


$\Lambda = 3$



$\Lambda = 0$

Observation



(Courtesy F. Sabou)

A Systematic Approximation

Exact in $\Lambda \to \infty$ limit and often accurate for $\Lambda = 3$

A Conservative Approximation

GCE2 conserves total angular momentum, energy, and enstrophy in absence of forcing and damping.

Realizable

GCE2 = Closed under selected triads = Realizable

Seamless

But: Slow

Curse of Dimensionality

The "Curse of Dimensionality" — How to Address?

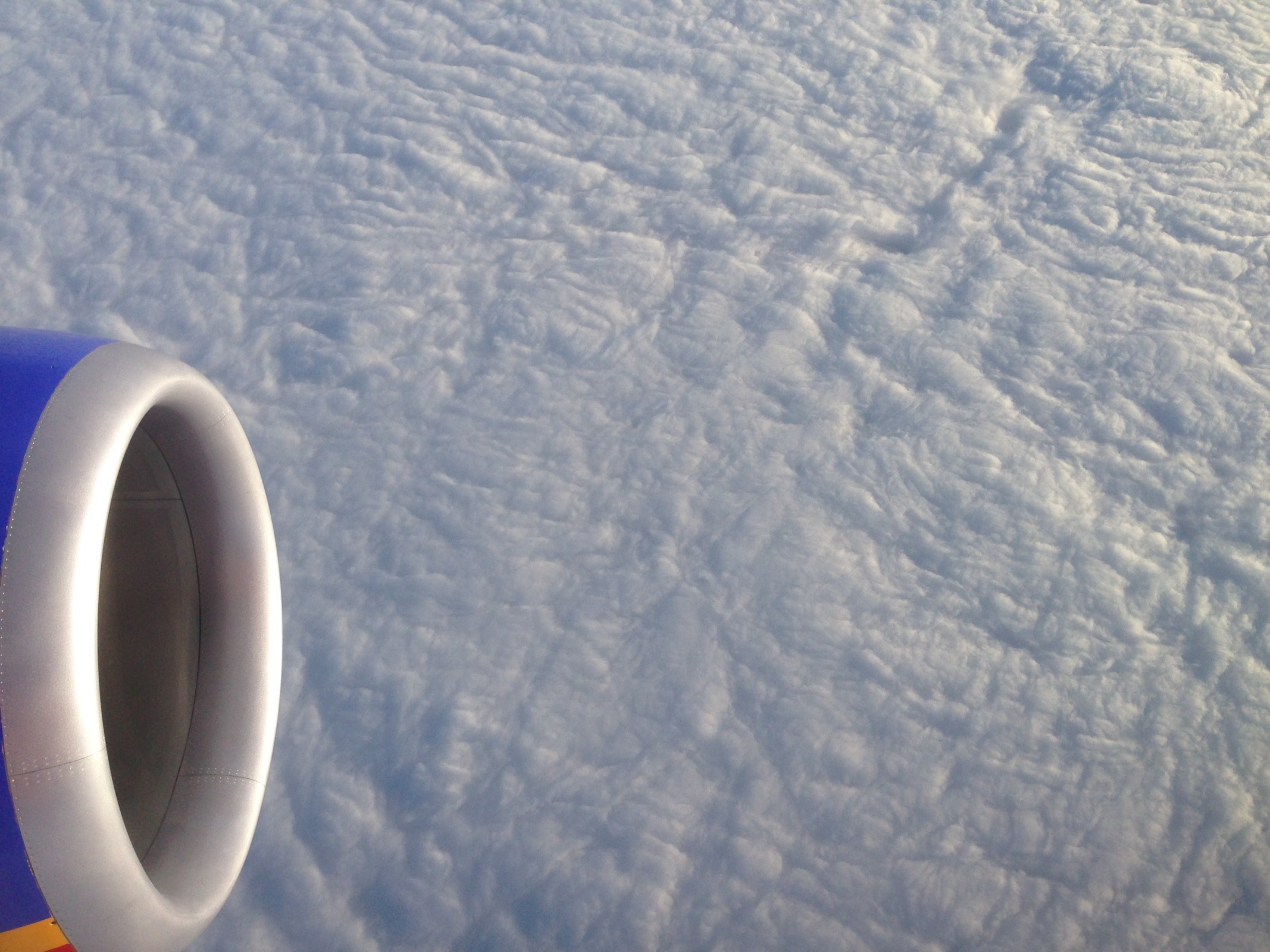
$$c(\vec{r}_1, \vec{r}_2) \equiv \langle h(\vec{r}_1) h(\vec{r}_2) \rangle$$

$$c(\vec{r}_1, \vec{r}_2) = \sum_i \lambda_i \; \phi_i(\vec{r}_1) \; \phi_i(\vec{r}_2)$$

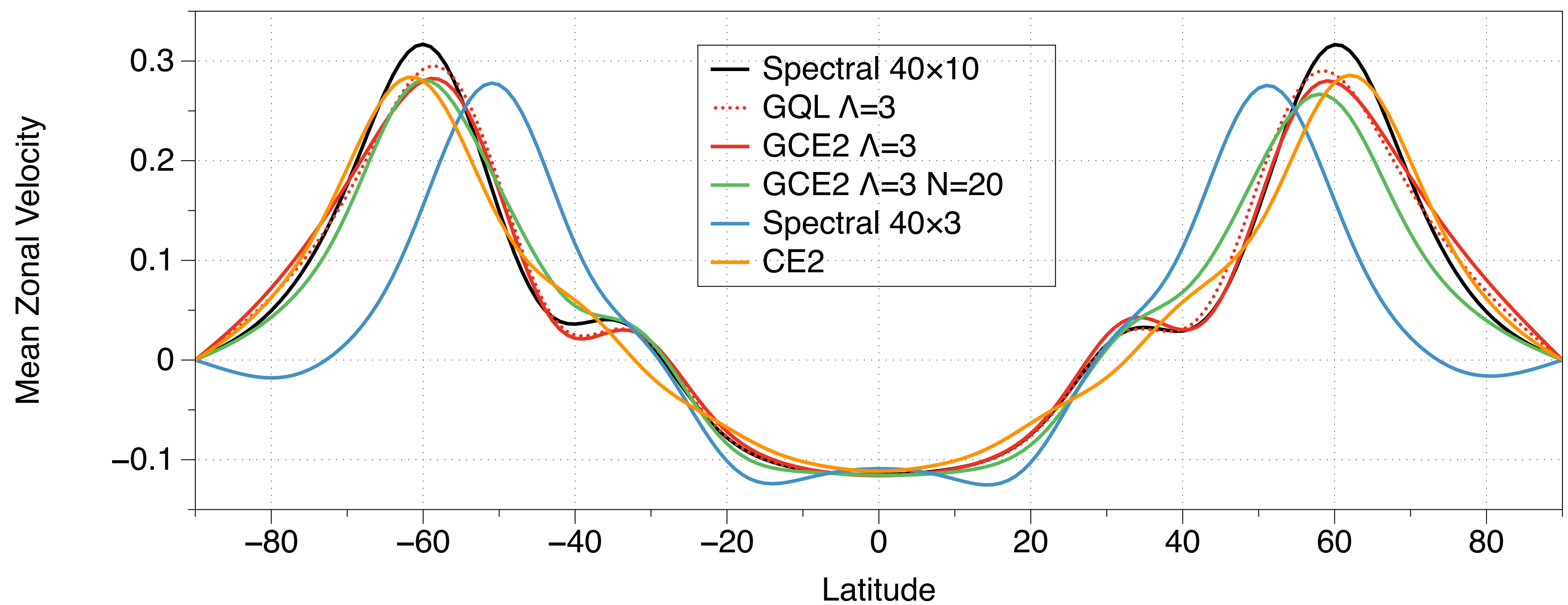
Schmidt decomposition

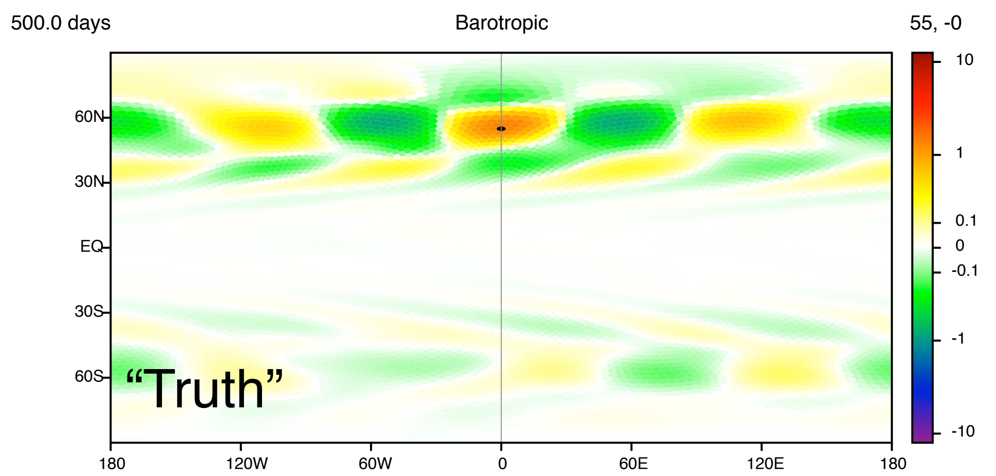
$$\approx \sum_{i=1}^{N} \lambda_i \; \phi_i(\vec{r}_1) \; \phi_i(\vec{r}_2)$$

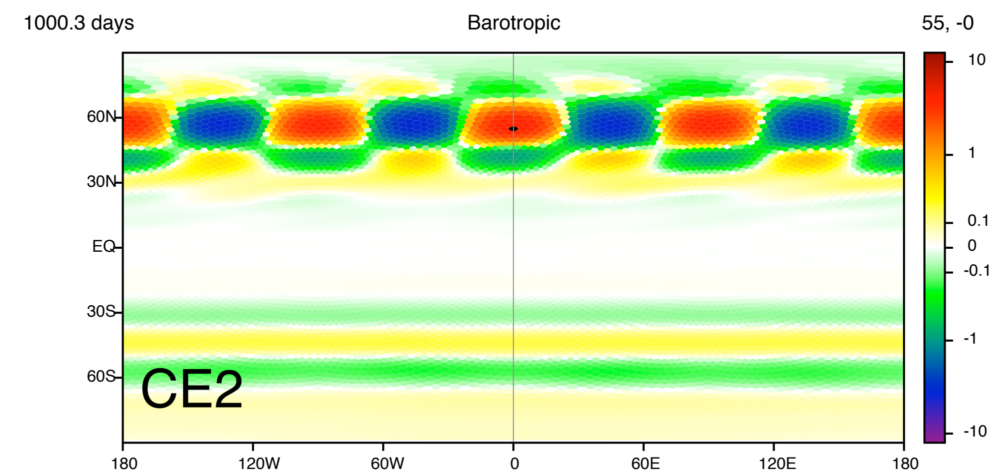
Entanglement:
More than one non-zero eigenvalue

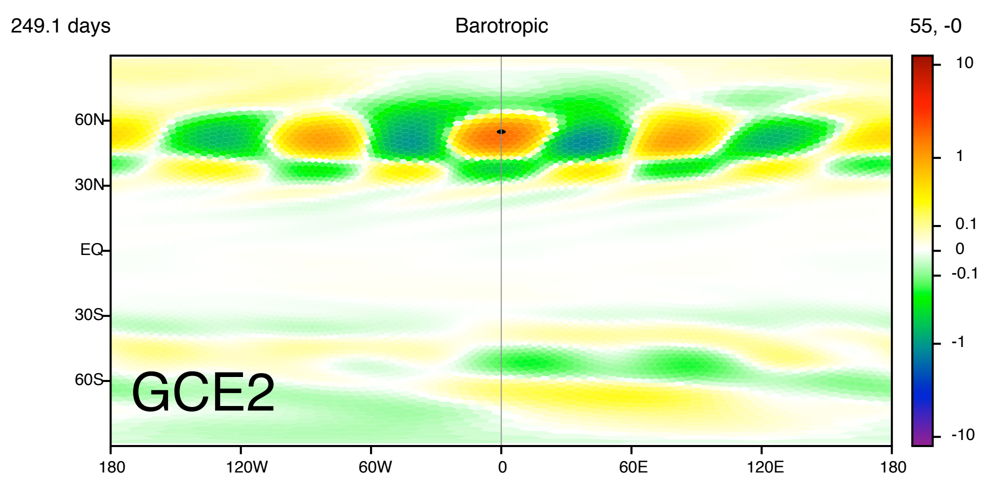


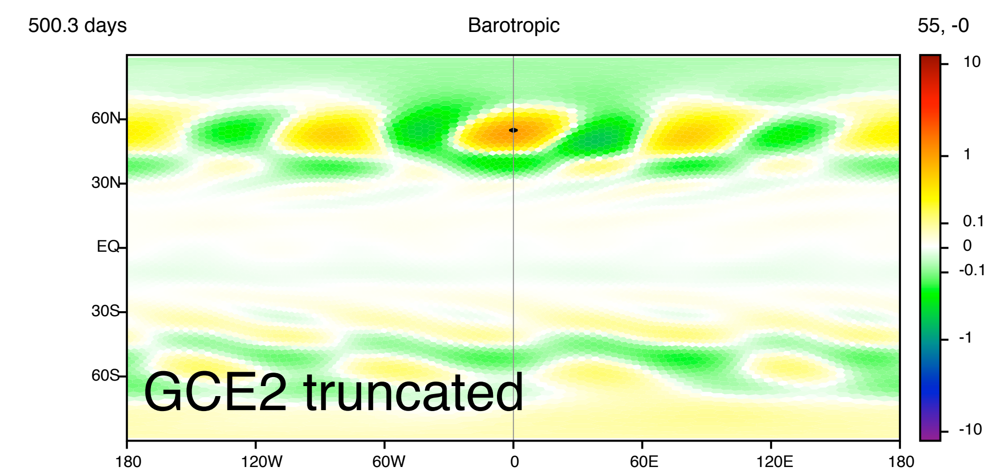
2-Layer Baroclinic QG Model





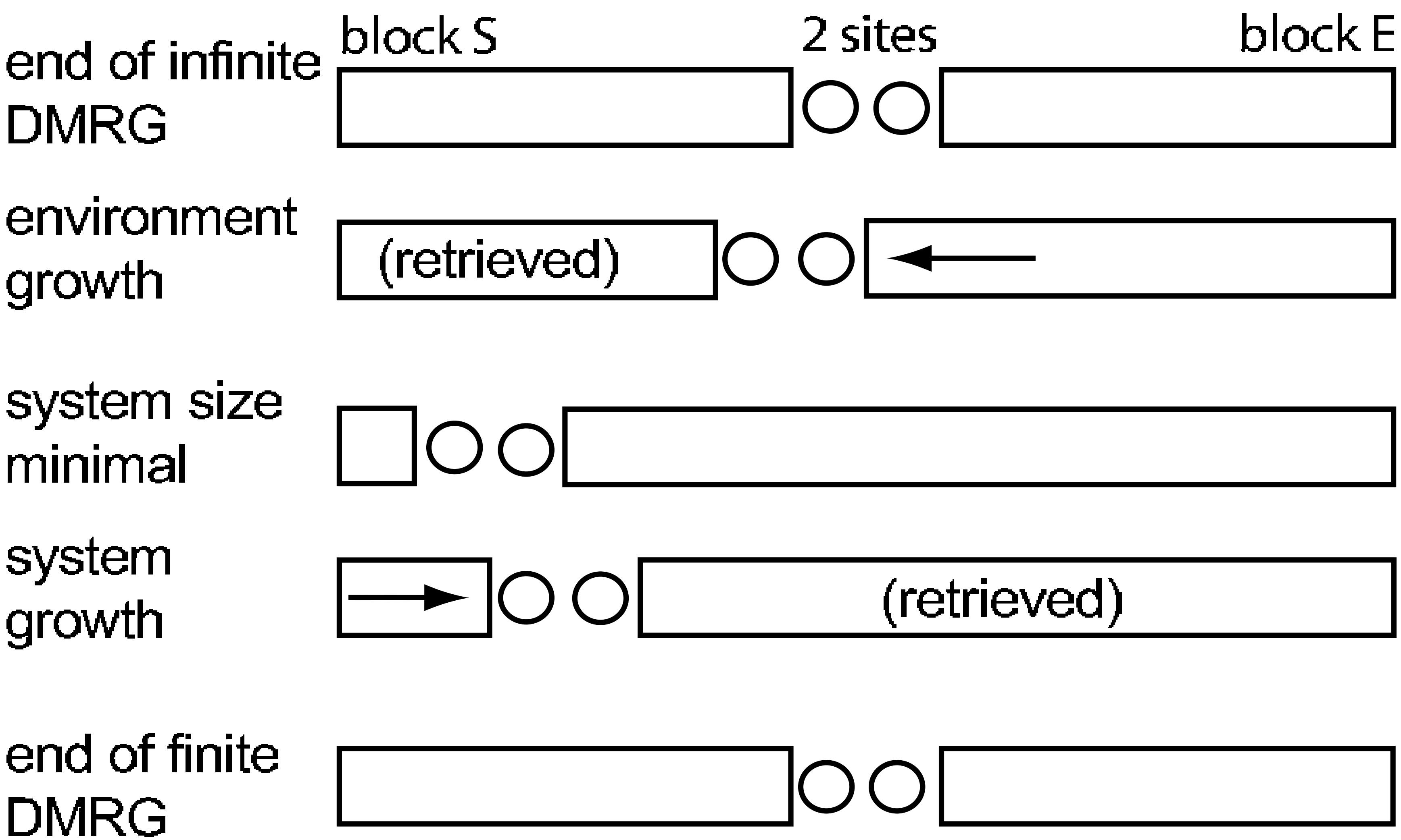






How to Update the Retained Basis Dynamically?

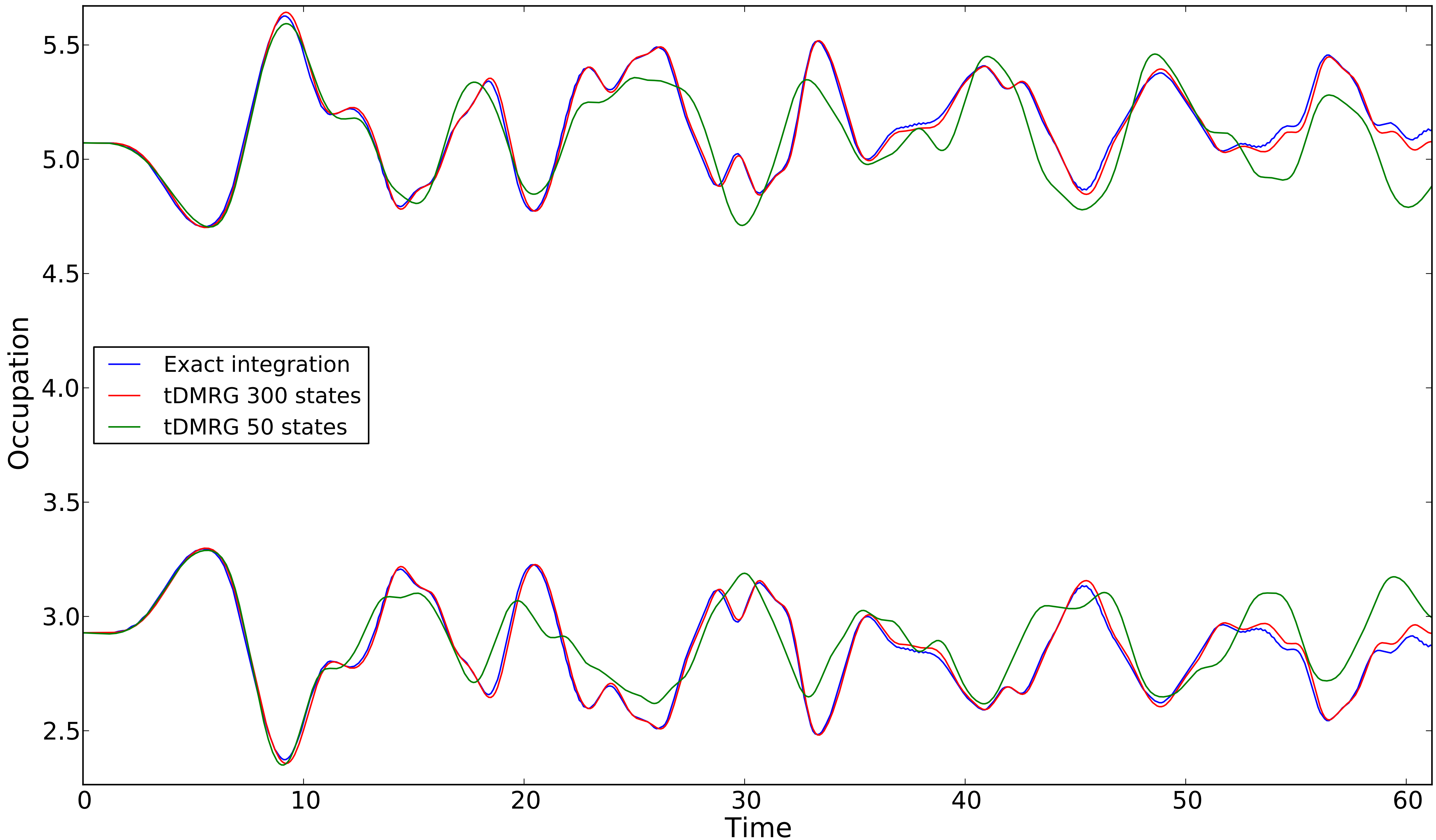
Density-Matrix Renormalization-Group (DMRG)



$$\dim(\mathcal{H}) = 2^N$$

Daley, Kollath, Schollwöck and Vidal (2012)

How to Update the Retained Basis Dynamically?



Sabou, Boddington, JBM (2012)

Open Questions

- How to dynamically adapt basis?
- Filter in real space, wavenumber space, or something else?

Thanks:

Grad Students

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Tom Iadecola
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