Dual constant-flux energy cascades in stably stratified rotating turbulence

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1: LASP; 2: NCAR; 3: Berkeley; 4: OakRidge

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Where does energy go?

* T-HI (N=0, f=0): Direct or *(exclusive)* inverse energy cascade

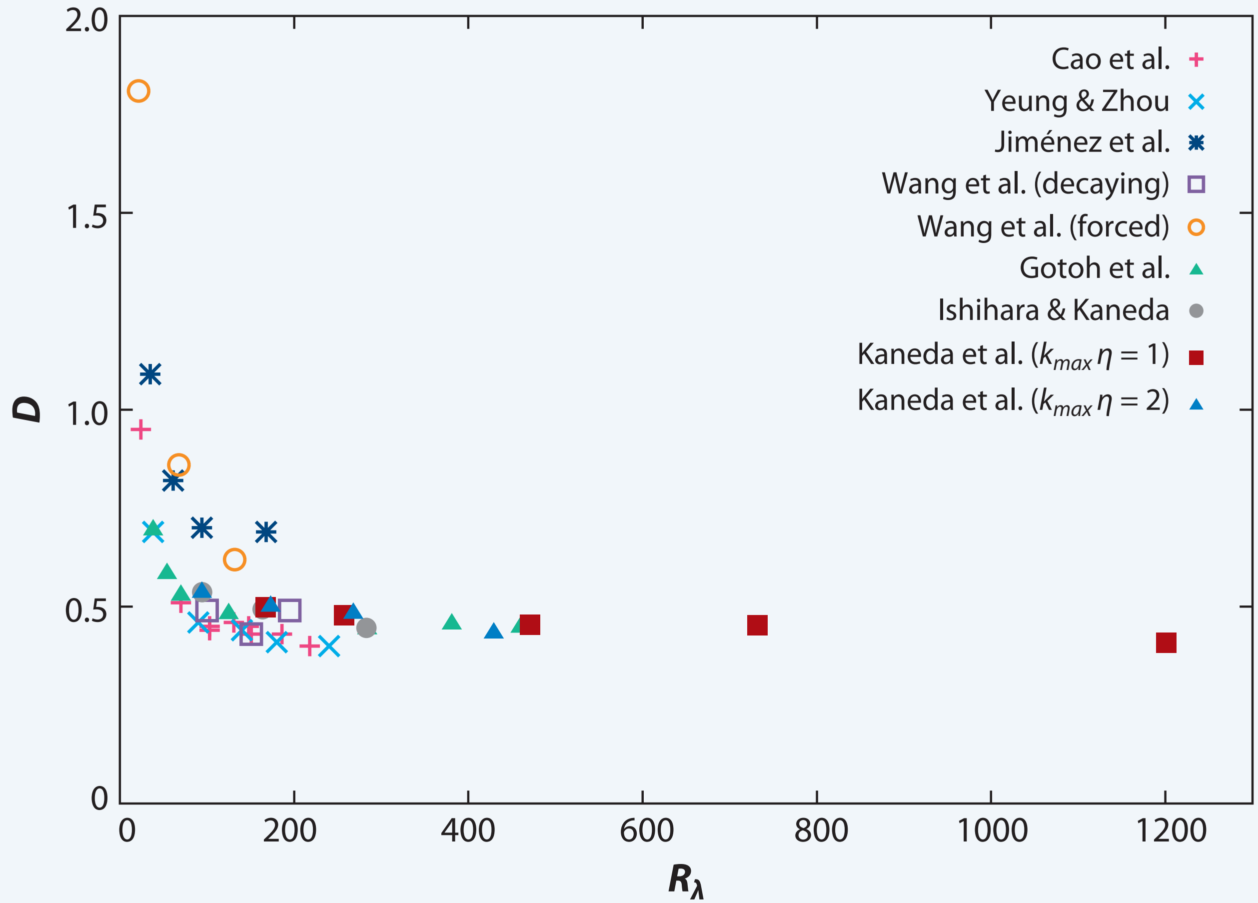
* Examples of dual bi-directional constant-flux cascades

* Oceanic data and an apparent paradox

* Direct numerical simulations: process study for a range of parameters

* Conclusions and questions

- Direct numerical simulations: process study for a range of parameters



Ishihara & Kaneda, Ann. Rev. 2009

Boussinesq equations

$$
\begin{aligned}
\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} &= -\nabla P - Nbe_z - 2\Omega e_z \times \mathbf{u} + \mathrm{F} \\
\partial_t b + \mathbf{u}\cdot\nabla b - \kappa\Delta b &= Nw \ , \\
\nabla\cdot\mathbf{u} &= 0 \ .
\end{aligned}
$$

Four dimensionless parameters: Re= UL/ν >> 1

Pr= ν/ϰ =1, Ro= U/[Lf] << 1 , Fr= U/[LN] << 1

R_B= Re Fr^2

2 ≤ N/f ≤ 10

f=2Ω

Boussinesq equations

$$
\begin{aligned}
\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} - \nu \Delta \mathbf{u} &= -\nabla P - N b e_z - 2\Omega e_z \times \mathbf{u} \; + \mathrm{F} \\
\partial_t b + \mathbf{u} \cdot \nabla b - \kappa \Delta b &= N w \; , \\
\nabla \cdot \mathbf{u} &= 0 \; .
\end{aligned}
$$

Four dimensionless parameters: $Re = UL/\nu \gg 1$
$Pr = \nu/\varkappa = 1$, $Ro = U/[Lf] \ll 1$, $Fr = U/[LN] \ll 1$

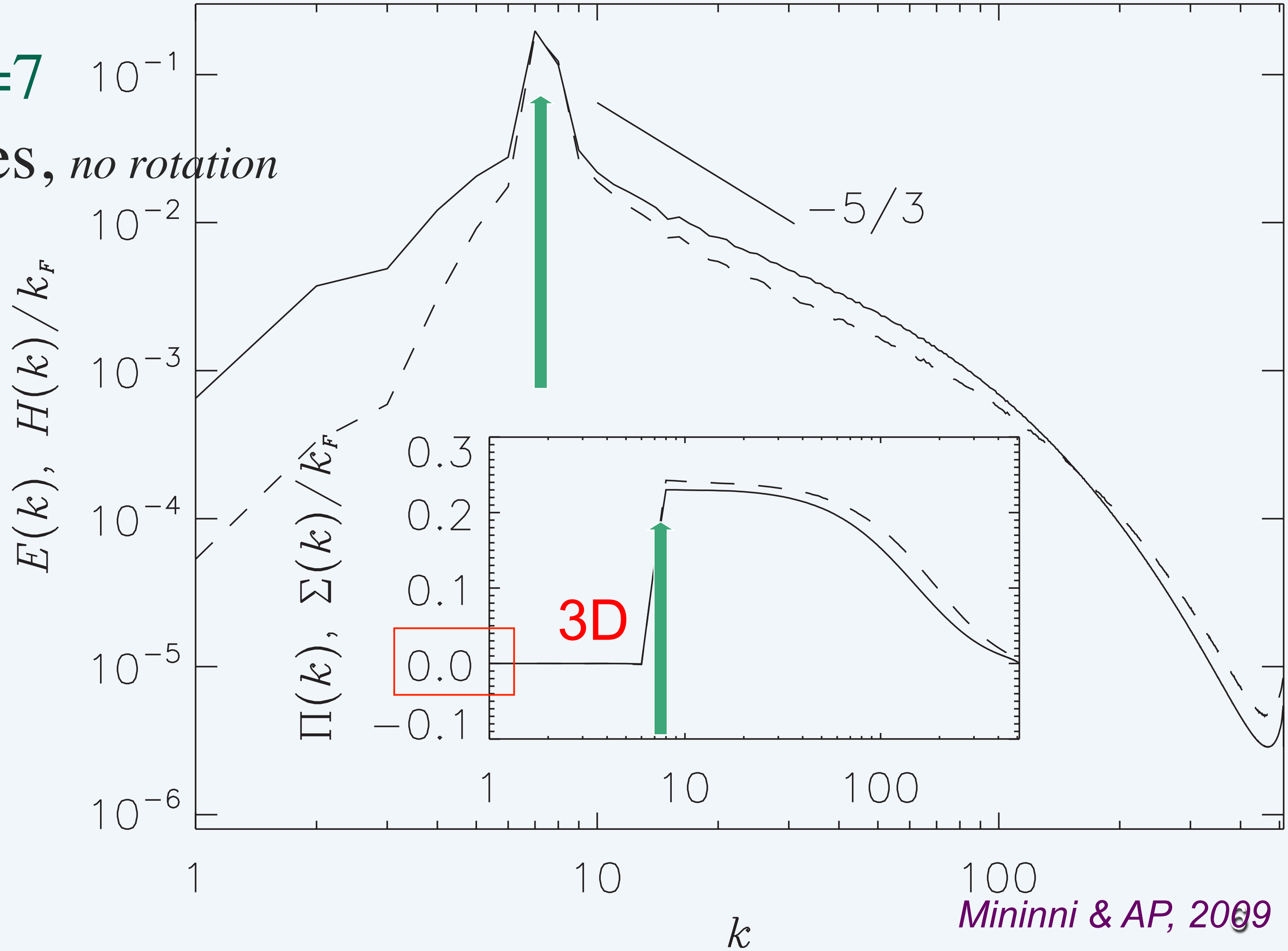
$R_B = Re\ Fr^2$
$2 \leq N/f \leq 10$

$f=2\Omega$

Spectra & fluxes for T-HI,
of energy ___
& helicity - - -
H= u.ω

Forced @ k=7

Navier Stokes, *no rotation*



Re=1200
1536^3 grid

Mininni & AP, 2009

2D

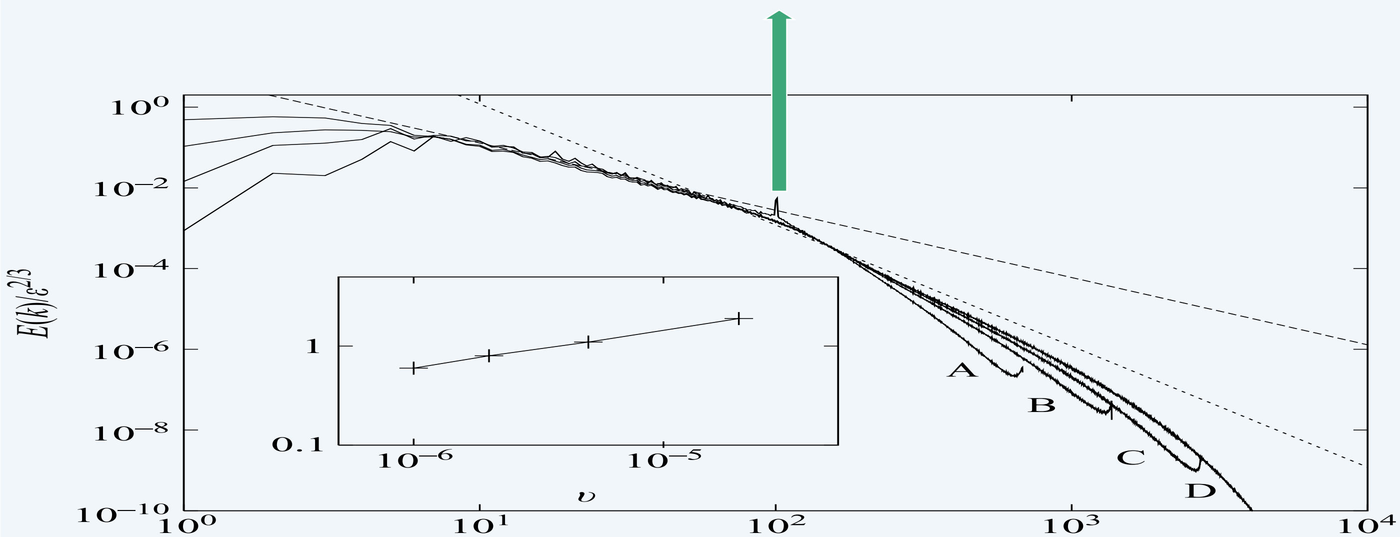
Forced @ k=100

* Friction

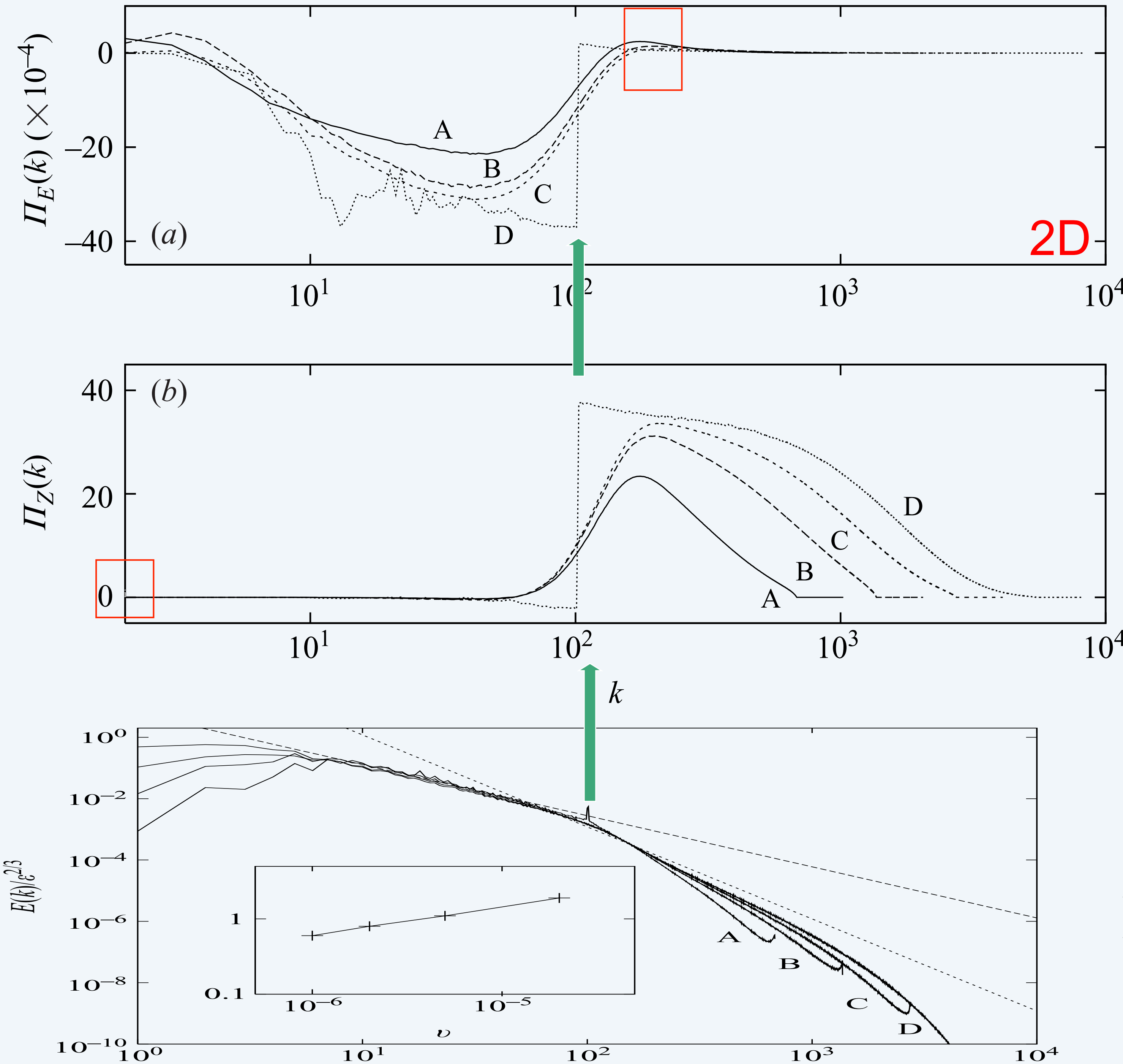
Grid up to 16384^2

$E(k)= C\,\varepsilon^{2/3}\,k^{-5/3}$, $C\sim6$

$E(k) \sim k^{-(3+x)}$



Boffetta 2007



Flux of **Energy**

and flux of

Enstrophy

Forced @ k=100

** Friction*

Grid up to 16384²

E(k)= C $\varepsilon^{2/3} k^{-5/3}$*, C~6*

E(k) ~ $k^{-(3+x)}$

Boffetta 2007

Paradigm with 2 invariants like energy & enstrophy:

2D: Dual but mutually exclusive system with an
inverse cascade of energy & a direct enstrophy cascade

3D: Direct cascade of energy, and direct helicity cascade

BUT …

3D, T-HI, 2D2C force, $A=L_z/L_x=1/64$ with $S = L_f/L_z$

Turbulent viscosity, Navier-Stokes, no rotation, 128^3 grid

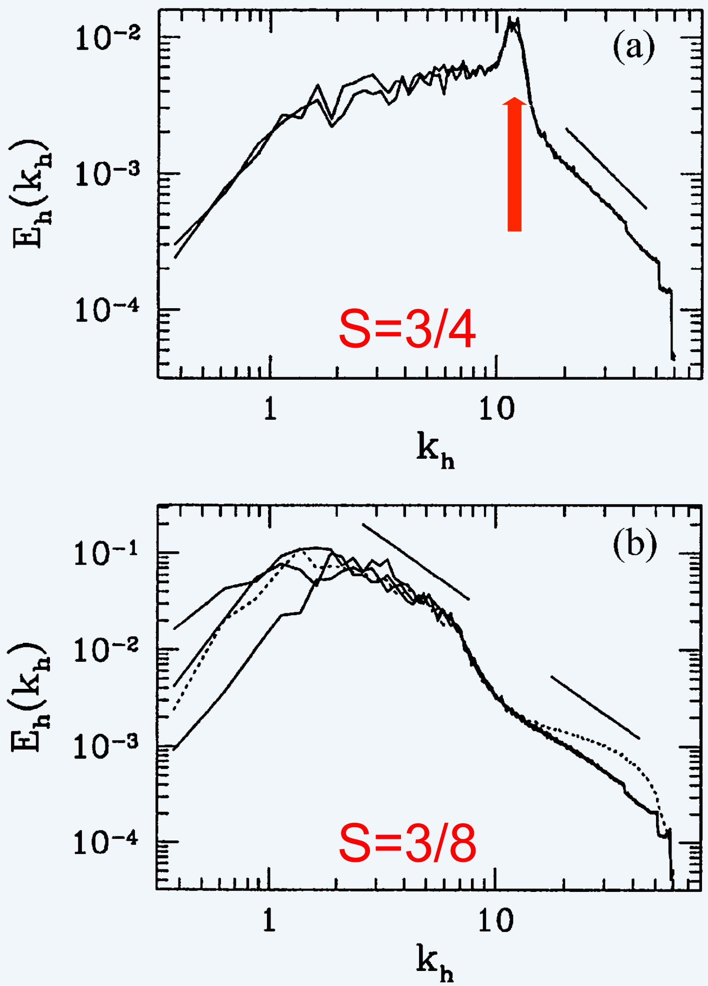
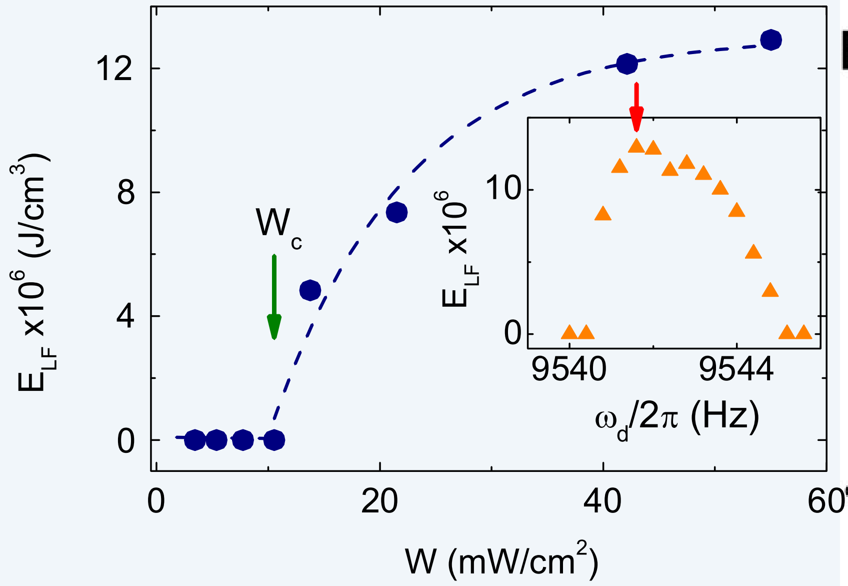


FIG. 2. (upper) $A = 1/64$, $\mathrm{Ro} = \infty$, $S = 0.75$ (statistically steady); (lower) $A = 1/64$, $\mathrm{Ro} = \infty$, $S = 0.375$: eddy viscosity (solid line) with time increasing upwards; hyperviscosity (dotted line). The lines are $E_h \propto k_h^{-5/3}$.

Smith et al. PRL 1996

(also: Celani et al. PRL 2010)

- Physical systems with dual cascades



Acoustic turbulence
in superfluid He

Energy in low frequency:
evidence for
an inverse cascade

Kolmakov et al. 2014,
After Ganshin et al. 2008

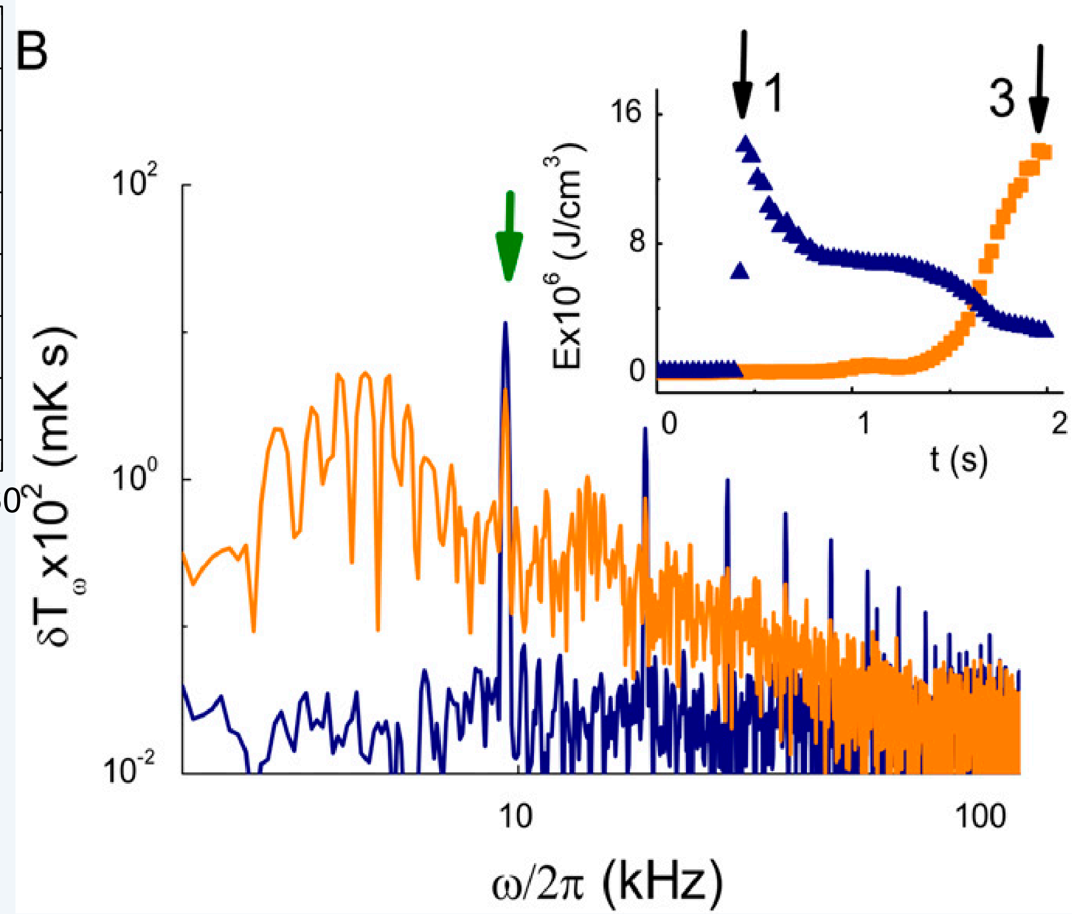
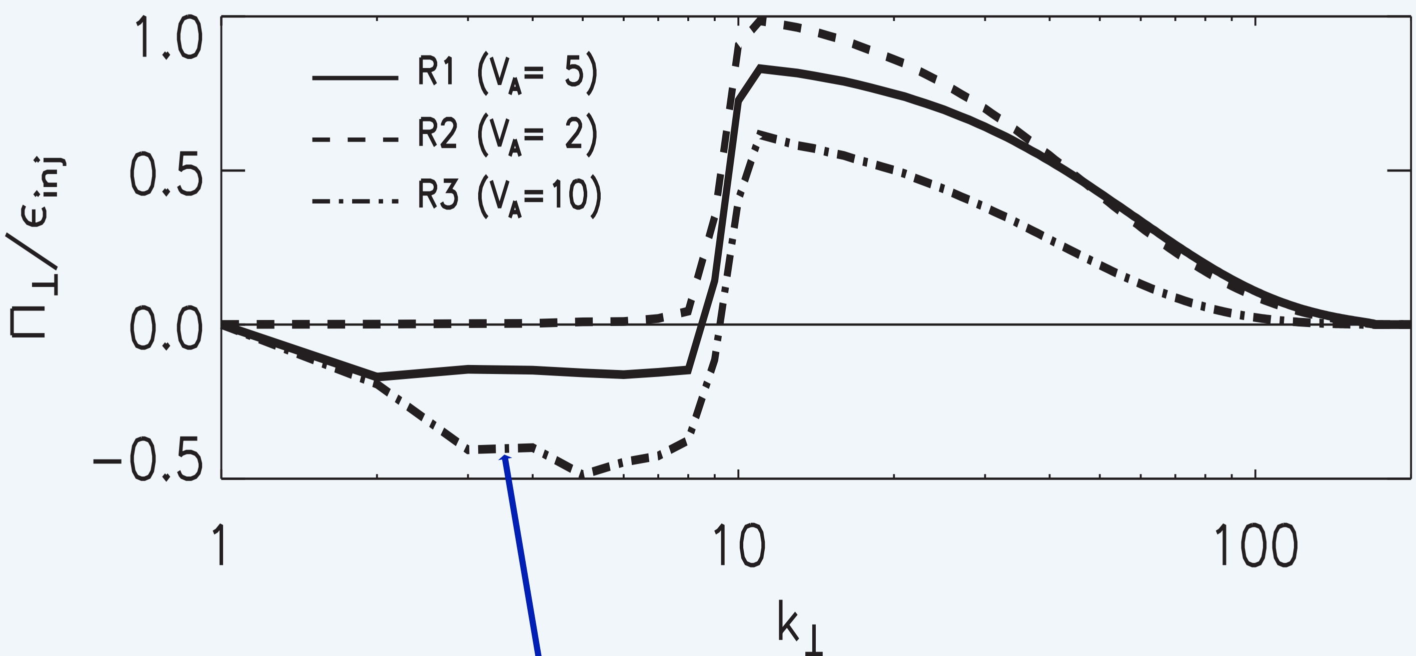


Fig. 3. (*A*) Transient evolution of the second sound wave amplitude δT after a step-like shift of the driving frequency to the 96th resonance at time t = 0.397 s. Formation of isolated "rogue" waves is clearly evident. (*Inset*) Example of a rogue wave, enlarged from frame 2. (*B*) Instantaneous spectra in frames 1 and 3 of *A*. The lower (blue) spectrum, for frame 1, shows the direct cascade only; the upper (orange) spectrum, for frame 3, shows both the direct and inverse cascades. The green arrow indicates the fundamental peak at the driving frequency. (*Inset*) Evolution of the wave energy in the low-frequency and high-frequency domains is shown by the orange squares and blue triangles respectively; black arrows mark the positions of frames 1 and 3. (After ref. 72.)

Kinetic energy flux in 3D MHD for various V_A



Stronger V_A

Alexakis, 2011

Energy flux in anisotropic shell models

PHYSICAL REVIEW E **83**, 066302 (2011)

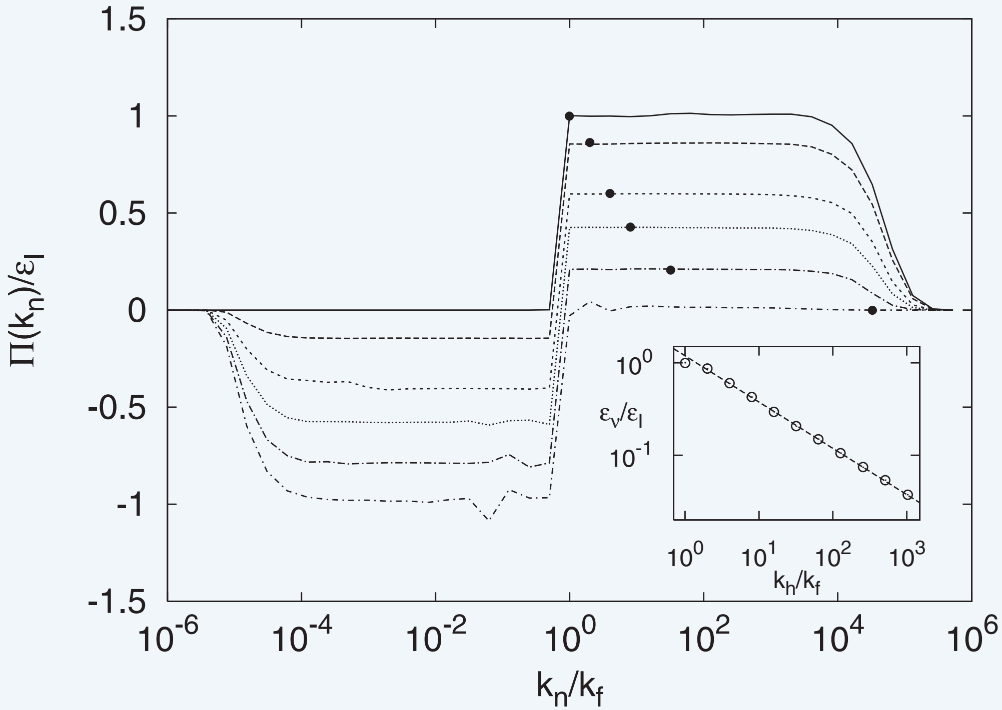


FIG. 2. Energy flux $\Pi(k_n)$ normalized with the input ε_I for increasing values of k_h/k_f from top to bottom: $k_h/k_f = 2^0, 2^1, 2^3, 2^5, 2^{15}$. The shell k_h is indicated by black dots on each curve. Inset: Energy flux in the direct energy cascade ε_ν as a function of the scale separation k_h/k_f. Dashed line represents the prediction $\varepsilon_\nu/\varepsilon_I \sim (k_h/k_f)^{-\beta}$.

Boffetta et al. 2011

- What happens
  with rotation
  and
  stratification
  in an idealized setting?

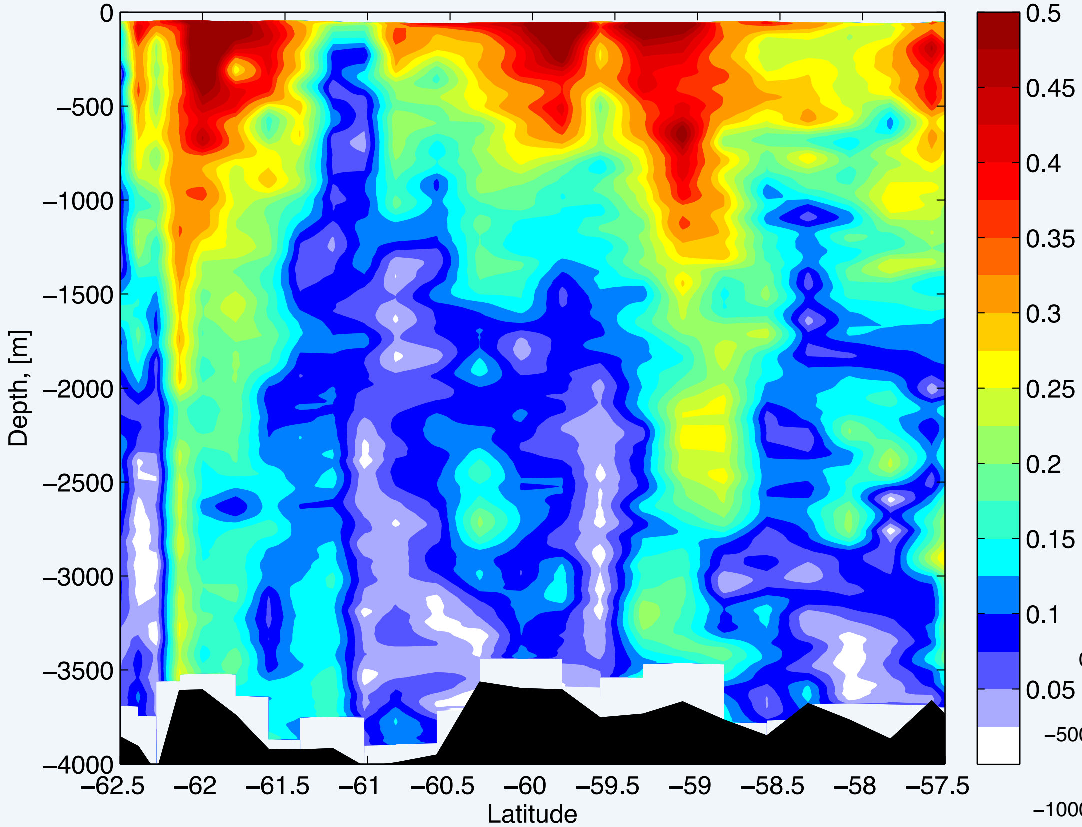


Figure 3-2: Flow speed (m s^{-1}) from the ALBATROSS section, Drake Passage.

Measurements in the Southern Ocean

← of flow speed

and of buoyancy frequency

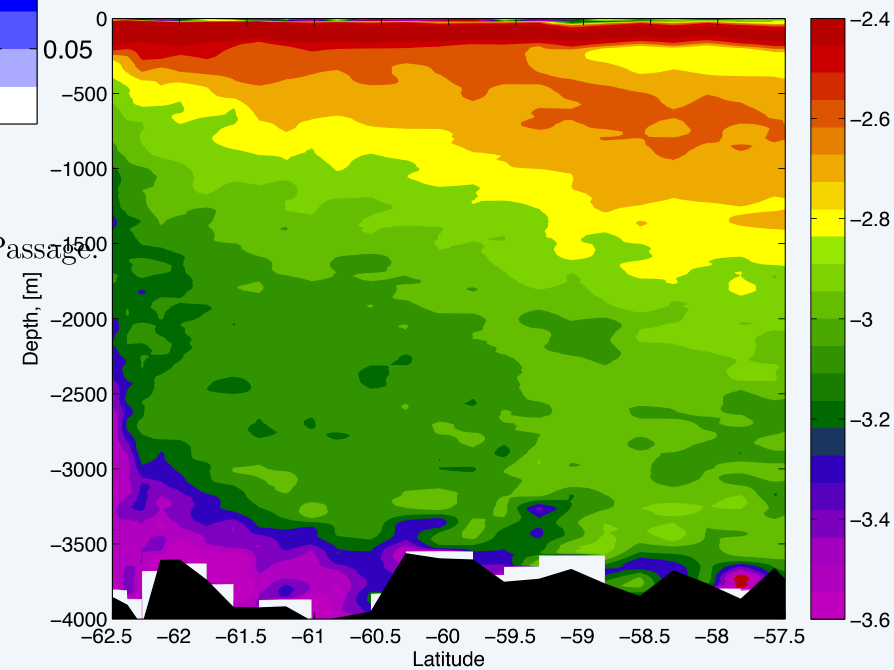
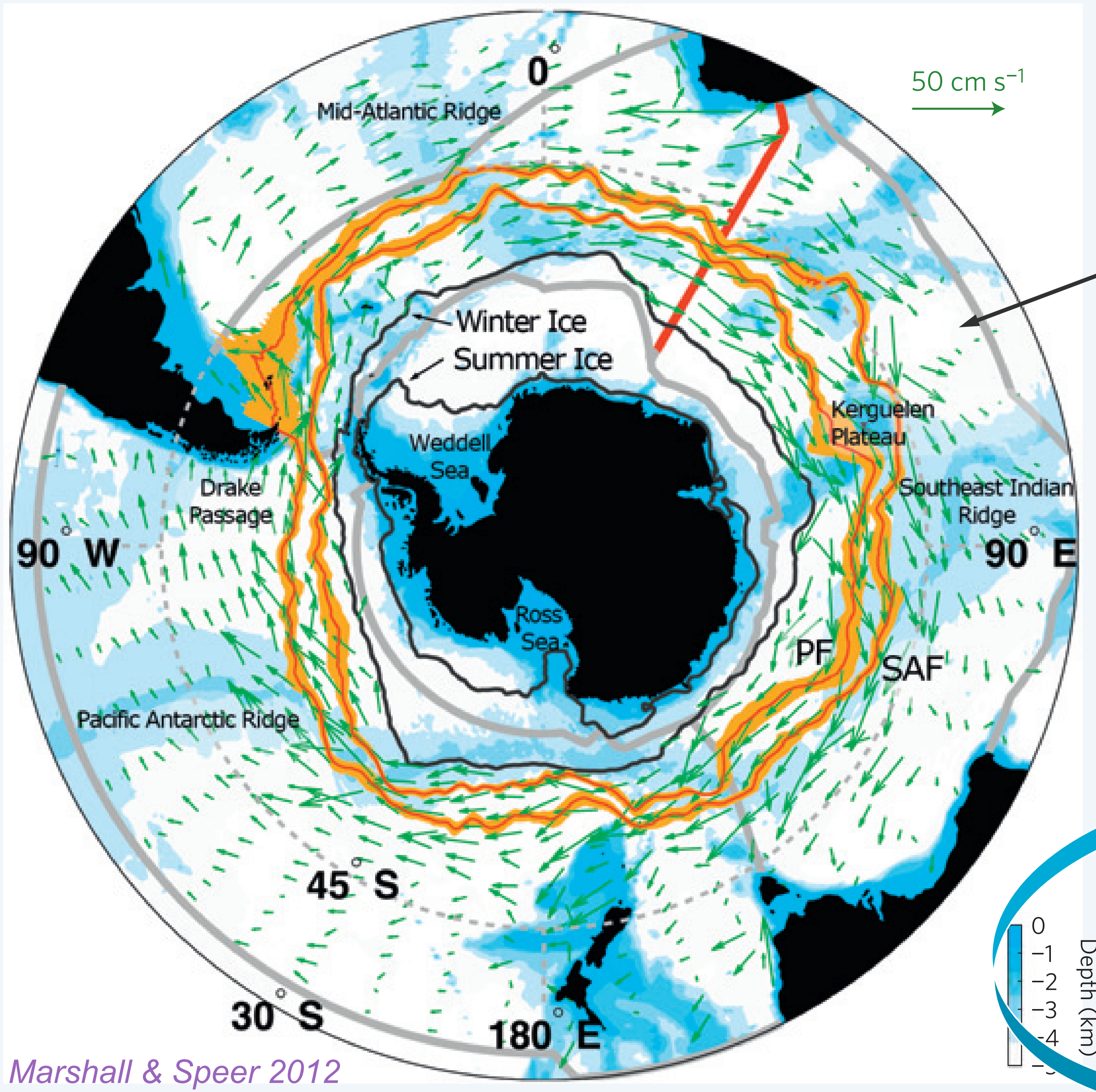


Figure 3-1: Buoyancy frequency (s^{-1}) in logarithmic scale from the ALBATROSS section, Drake Passage.

Nikurashin, 2009



Marshall & Speer 2012

5km³

next to the Kerguelen Plateau

$L_F \sim 500m$

$U \sim 0.04m/s$

$N=0.001\ /s$

$f = N / 10$

Fr= 0.08

Ro = 0.8

$\nu = 10^{-6}\ m^2/s$

$Re = 2.\ 10^7$

$R_B \sim 10^5$

DNS run

Boussinesq

2048³ grid

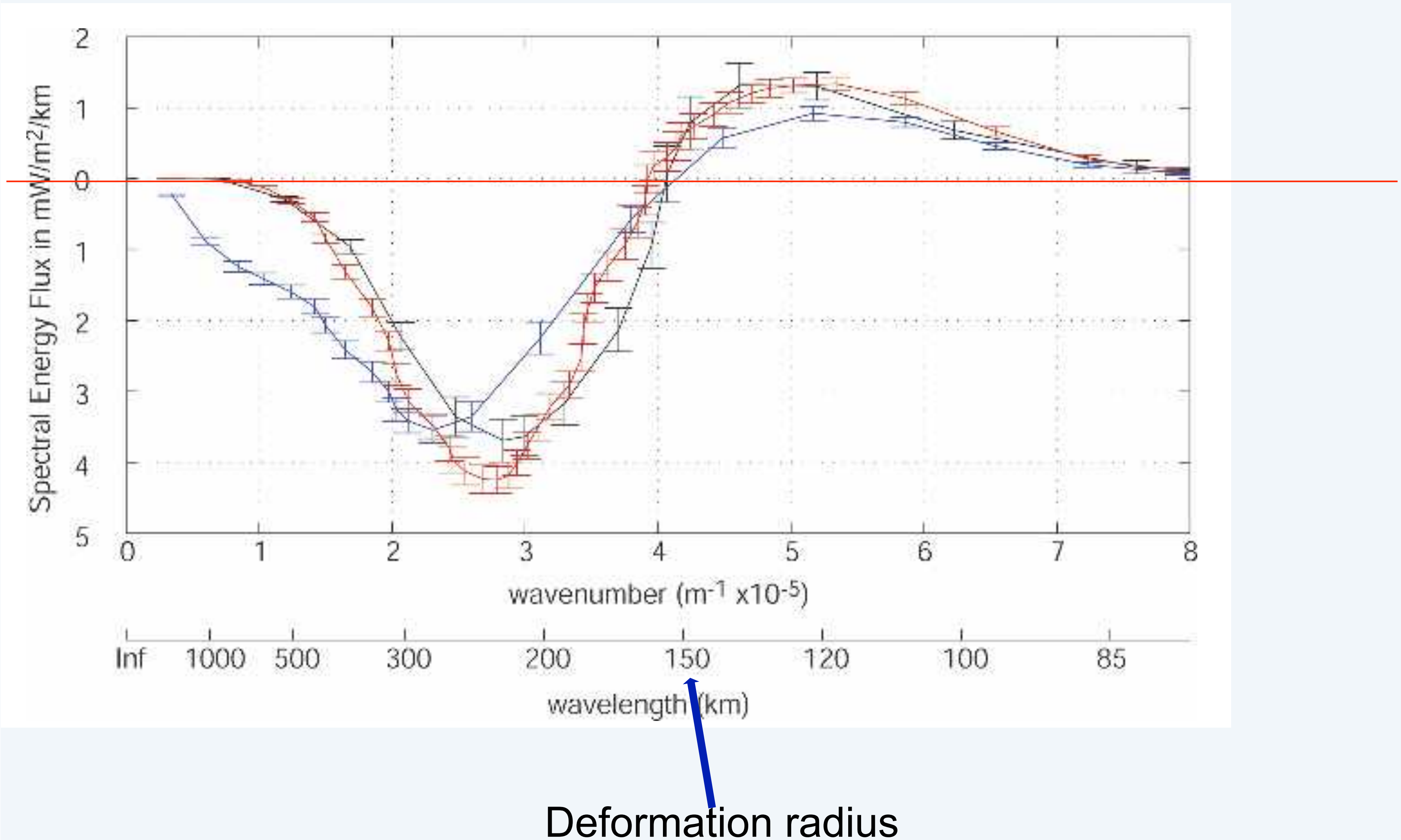
$\nu = 8x\ 10^{-4}\ m^2/s$

Re=24000

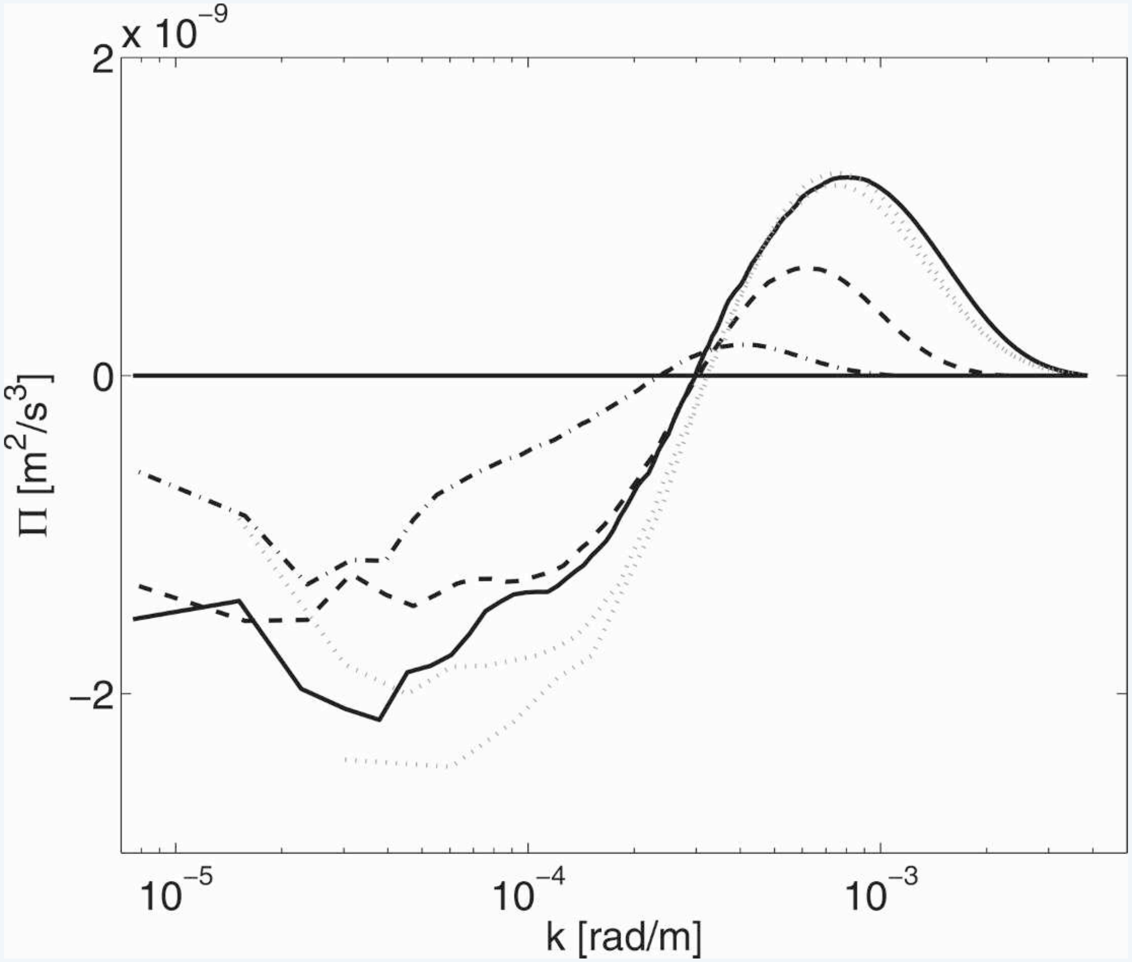
$R_B \sim 150$

$Pr = 1$

Kinetic energy flux in the ACC, 10+yrs data every 10 days ~ T_{NL}



Scott 2005



← Energy flux and spectrum

ROMS

Forcing in momentum, fresh water & heat with restoring force, KPP & sponge layer

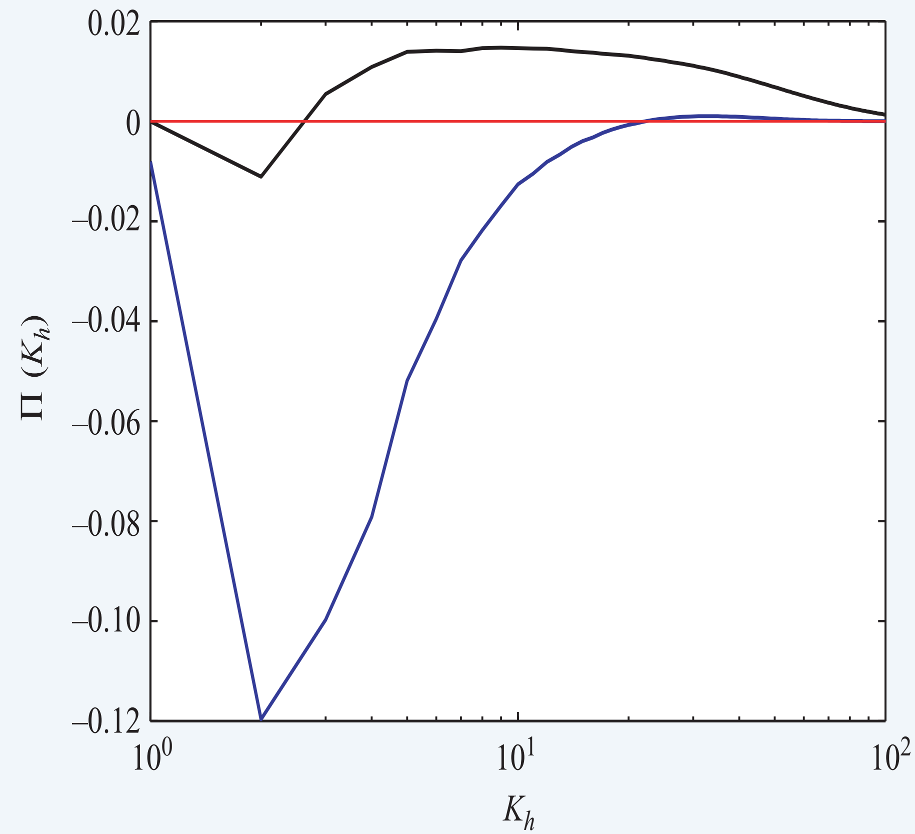
Down to 0.75km res.

(solid line)

Larger range for the inverse cascade than for the direct one

Capet et al., 2008

M. J. Molemaker, J. C. McWilliams and X. Capet



5. Spectral flux of kinetic energy ($Re_{eff} = 6600$): BOUS ($Ro_r = 0.5$, black) and QG (blue). Note the forward cascade for BOUS and inverse cascade for QG.

QG *(blue)*

Boussinesq *(black)* with shear

Energy flux

Molemaker et al. 2010

A paradox?

- *Capet et al. (2008), ROMS+KPP:*

… we hesitate to draw any strong conclusions about the efficacy of a mesoscale inverse KE *{Kinetic Energy}* cascade in our solutions, although our results indicate it does occur to some degree …

** Scott et al. (2011), oceanic data analysis:*

> … despite great effort in studying the ocean's energy budget in the last two decades, the bulk of the dissipation of the most energetic oceanic motions remains unaccounted for.

** Arbic et al. (2013), oceanic data and modeling:*

… It is therefore difficult to say whether the forward cascades seen in present-generation altimeter data are due to real physics (represented here by eddy viscosity) or to insufficient horizontal resolution.

Geophysical High Order Suite for Turbulence *(Gomez & Mininni)*

- Pseudo-spectral DNS, periodic BC cubic *(also 2D), single/double precision*; Runge-Kutta for incompressible Navier-Stokes, SQG & Boussinesq. Includes rotation, passive scalar(s), MHD + Hall term
- GHOST, from laptop to high-performance, parallelizes linearly up to 100,000 processors, using hybrid MPI/Open-MP (Mininni et al. 2011, Parallel Comp. 37)
- 3D Visualization: VAPOR (NCAR); and development @ OakRidge (D. Rosenberg)
- LES: alpha model & variants (Clark, Leray) for fluids & MHD
- Helical spectral (EDQNM) model for eddy viscosity & eddy noise
- *NEW!* Lagrangian particles *(w. A. Pumir, ENS)*
- *NEW!* Gross-Pitaevskii & Ginzburg-Landau *(with M. Brachet, ENS)*
- ***Data**, forced: 2048^3 Navier-Stokes and 1536^3 & 3072^3 with rotation, both w. or w/o helicity. Rotating stratified turbulence w. 2048^3 grids.*
- *Spin-down MHD:1536^3 random + 6144^3 ideal & 2048^3 w. T-Green symmetry.*
- *Decaying rotating stratified flow, N/f~5, Re=5.5 10^4, 2048^3, 3072^3 & 4096^3 grids*

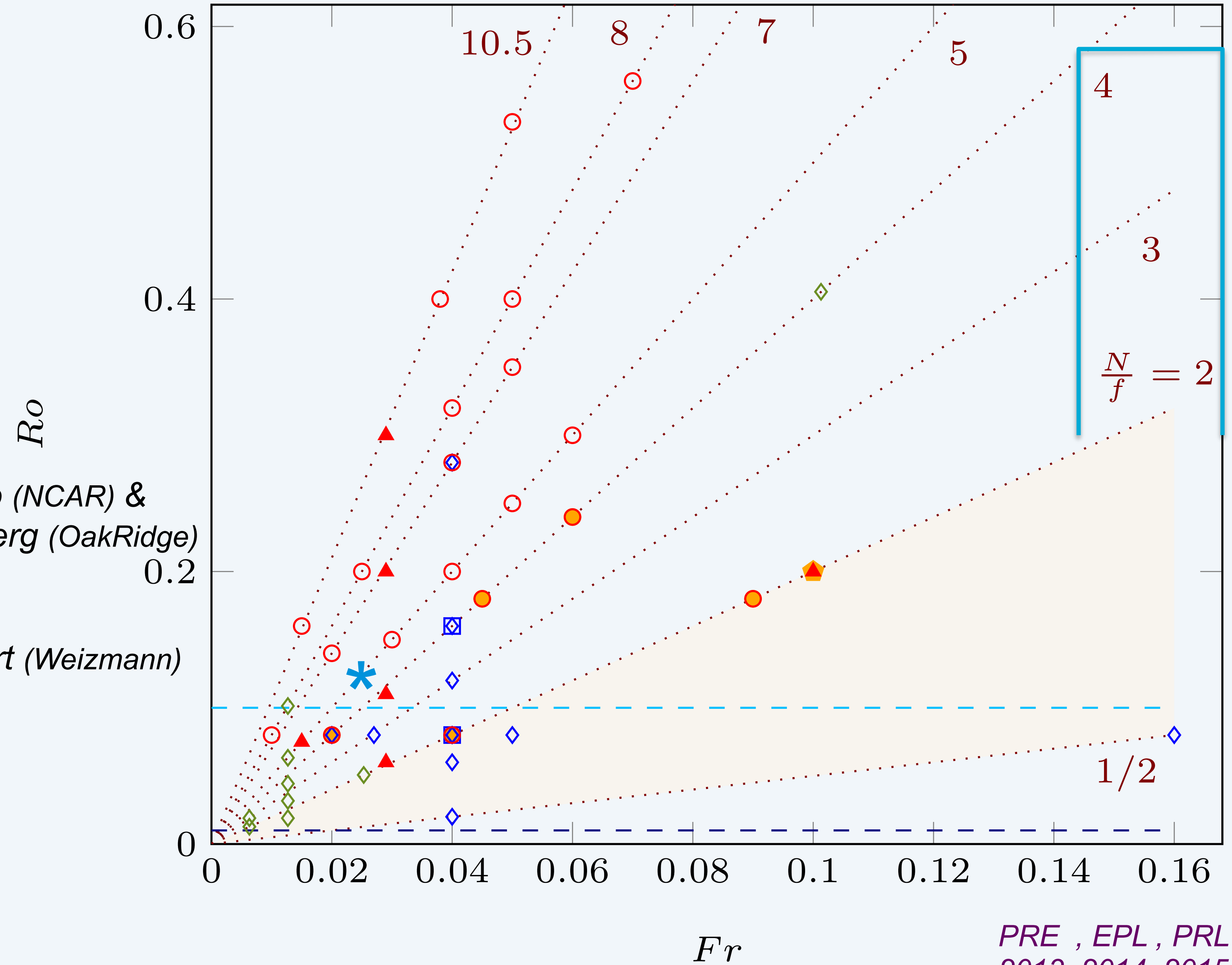
mininni@df.uba.ar duaner62@gmail.com

Rotating-stratified data

Runs:

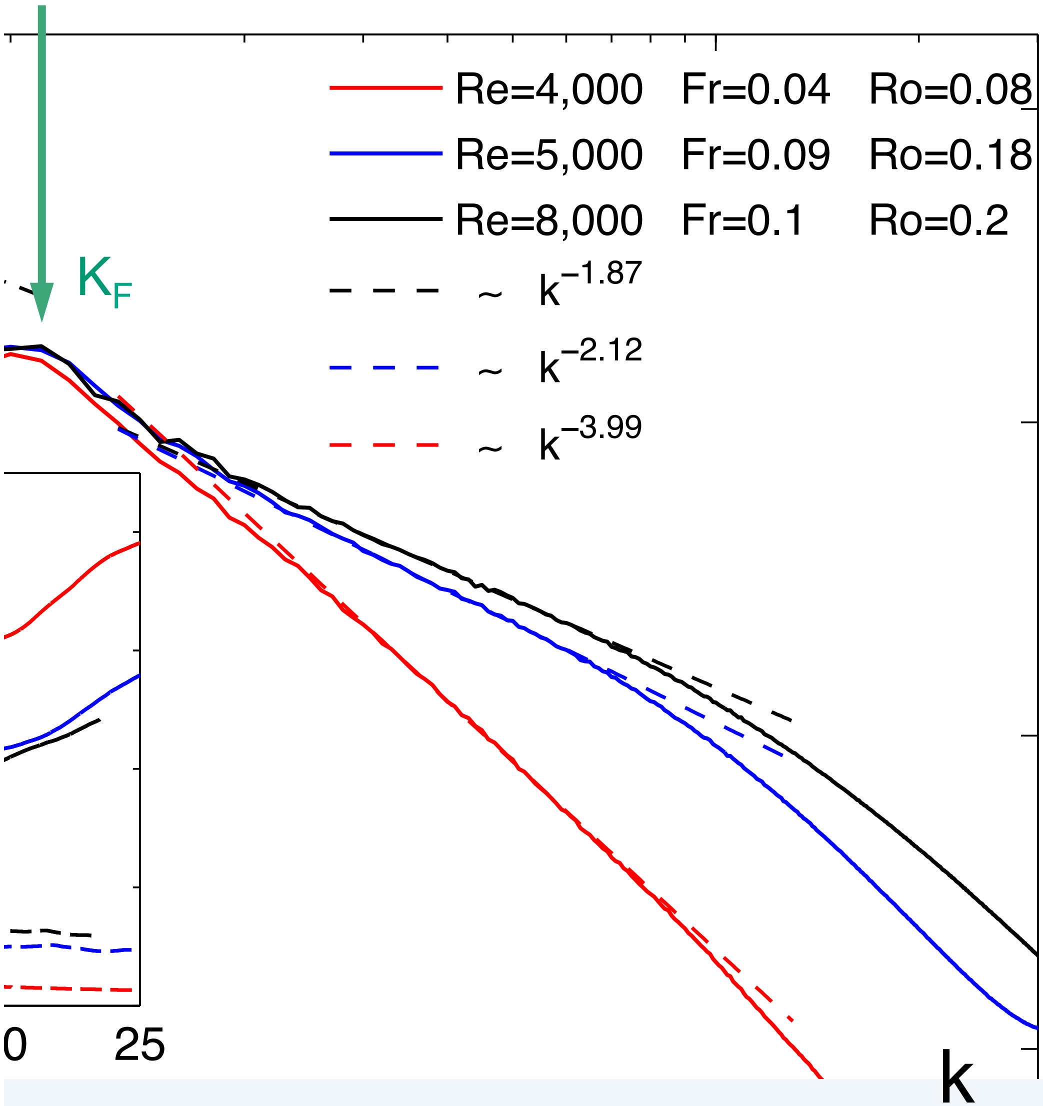
Raffaele Marino (NCAR) &
Duane Rosenberg (OakRidge)

Diagram:
Corentin Herbert (Weizmann)



PRE , EPL , PRL
2013, 2014, 2015

CPU: NSF (XSEDE & Yellowstone/NCAR); * : DOE/INCITE, 4096^3 grid



Forcing at $K_F \sim 10$

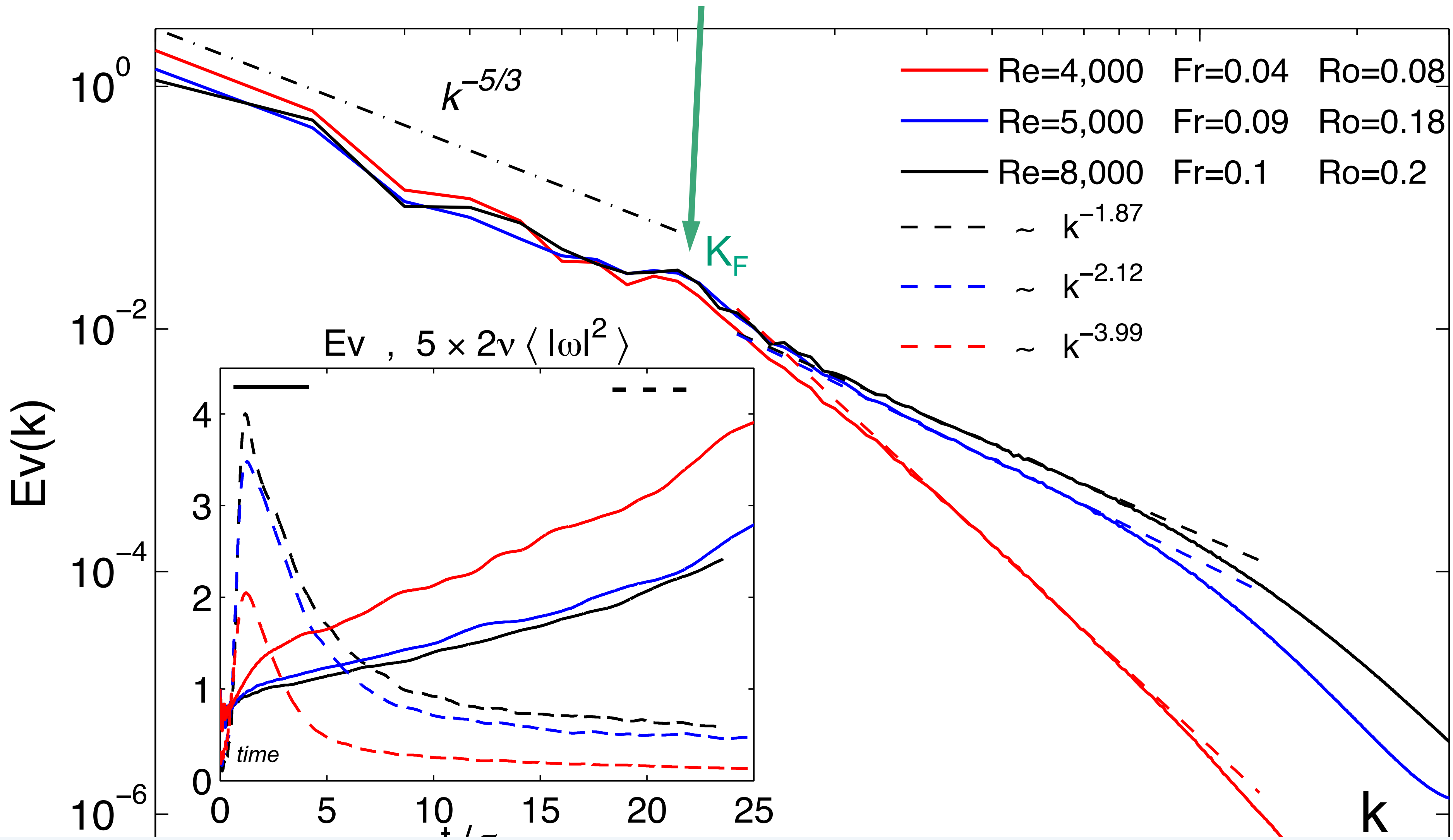
Small-scale spectra N/f =2 and for different parameters

R_B= Re Fr^2

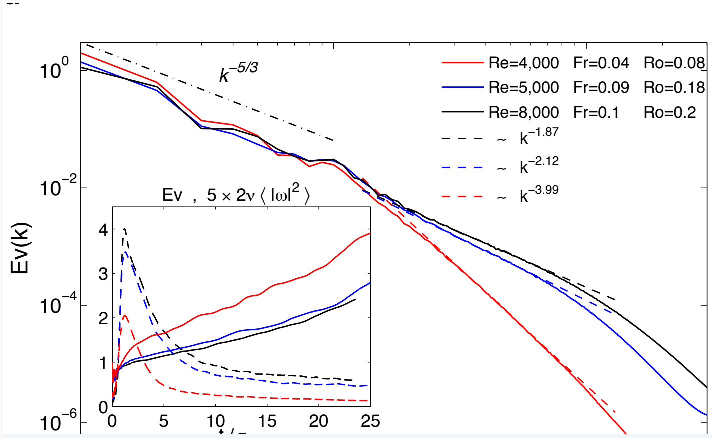
R_B= 6 , 40 & 80

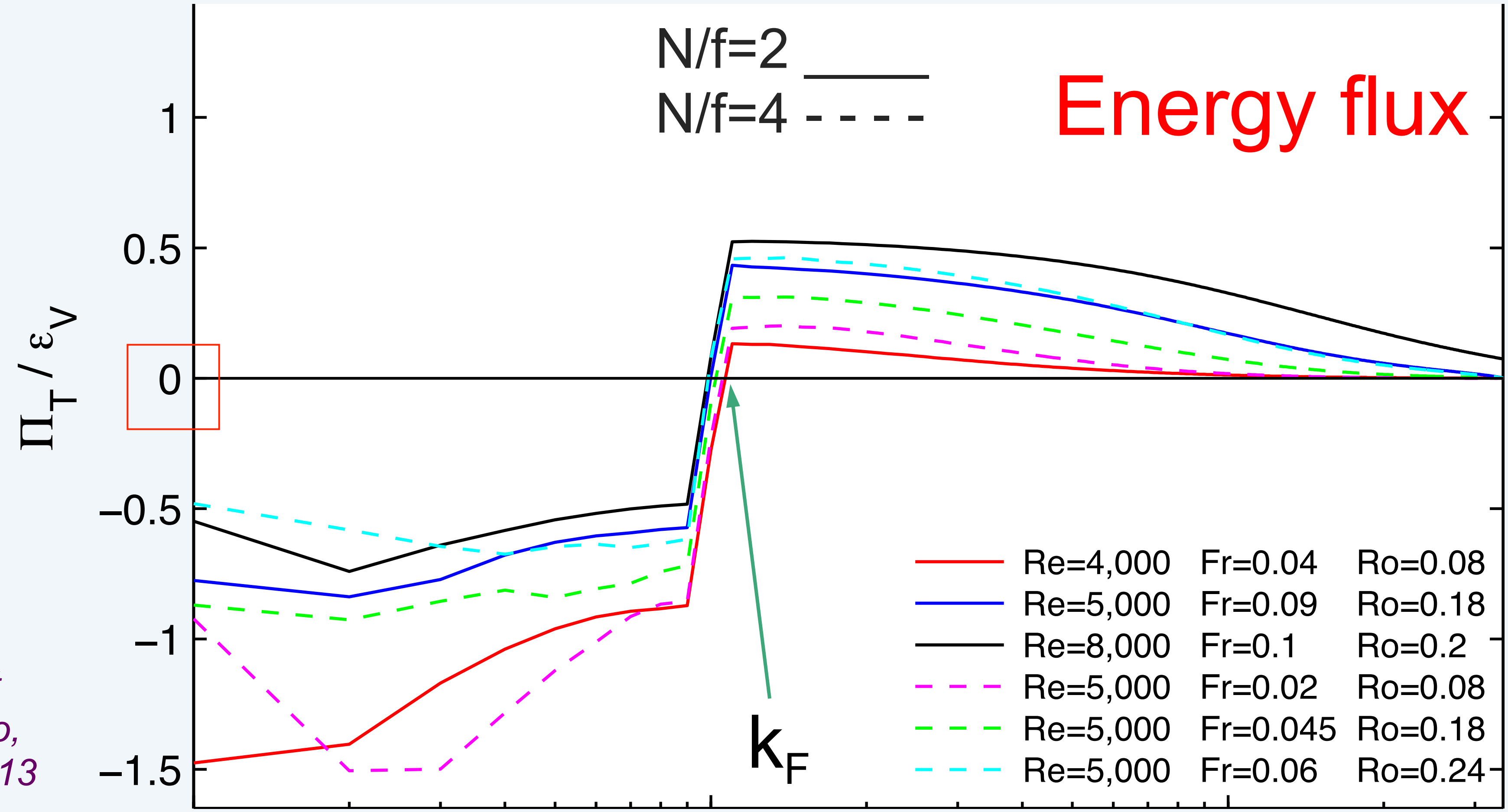
R_B= 120, E(k) ~ $k^{-1.77}$

Large-scale spectra, N/f=2



Temporal growth of energy (____) & stabilisation of energy dissipation (- - - -)





Pouquet & Marino, PRL 2013

Grids of 1024^3, 1536^3 & 2048^3 points, K_F= [10,11]

Run	Re	Fr	Ro	N/f	$\mathcal{R}_B$	R_Π	α
10a	5000	0.020	0.08	4	2.0	5.77	-3.99
10b	5000	0.045	0.18	4	10.1	2.70	-2.93
10c	5000	0.060	0.24	4	18.0	1.36	-2.34
10d	4000	0.040	0.08	2	6.4	9.04	-3.99
10e	5000	0.090	0.18	2	40.5	1.62	-2.12
15a	8000	0.100	0.20	2	80.0	1.08	-1.87
***20a**	12000	0.1	0.2	2	120	1.05	-1.77

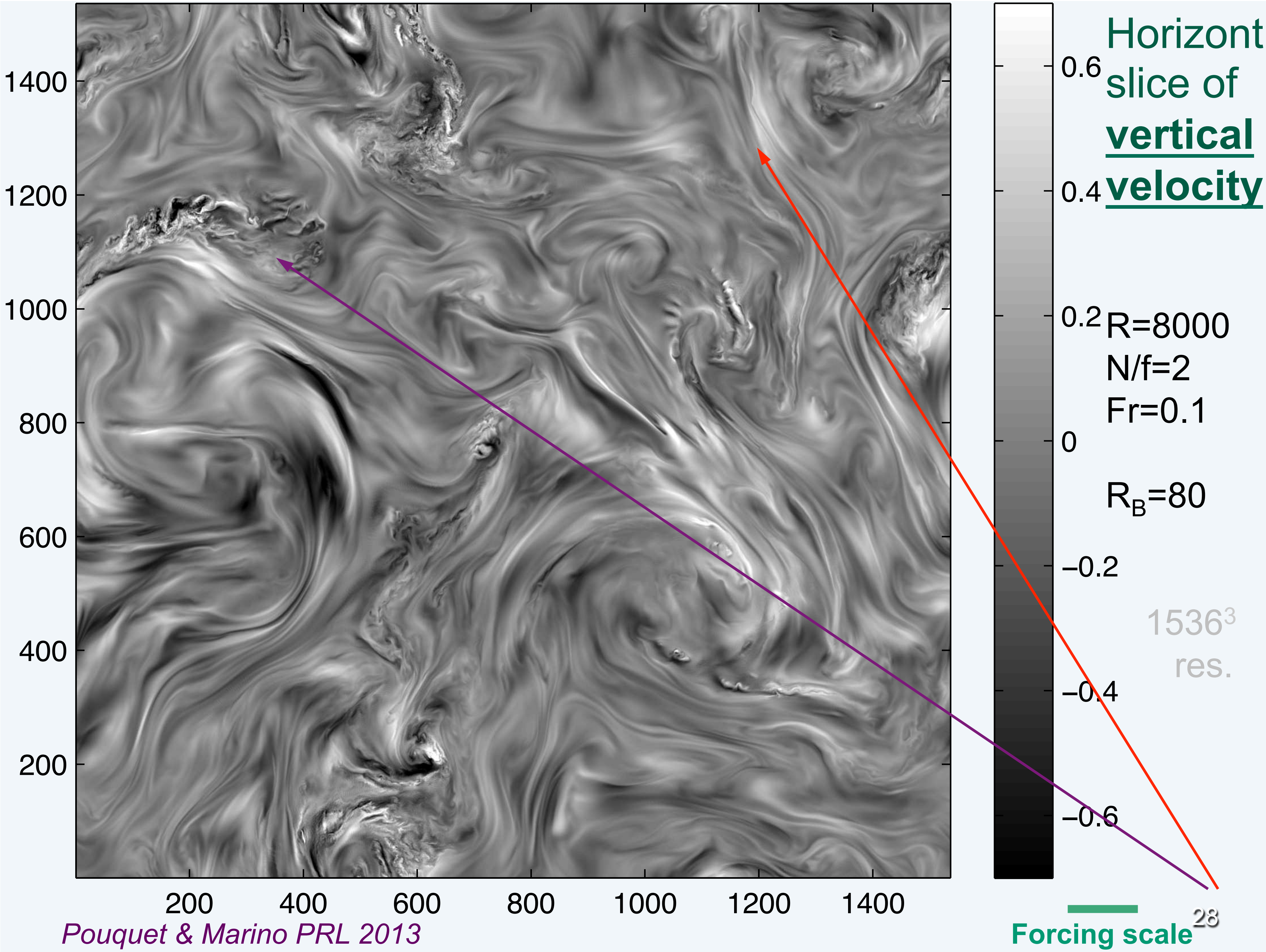
Re=UL/ν, Fr=U/[LN], Ro=U/[Lf]

$R_B = ReFr^2$

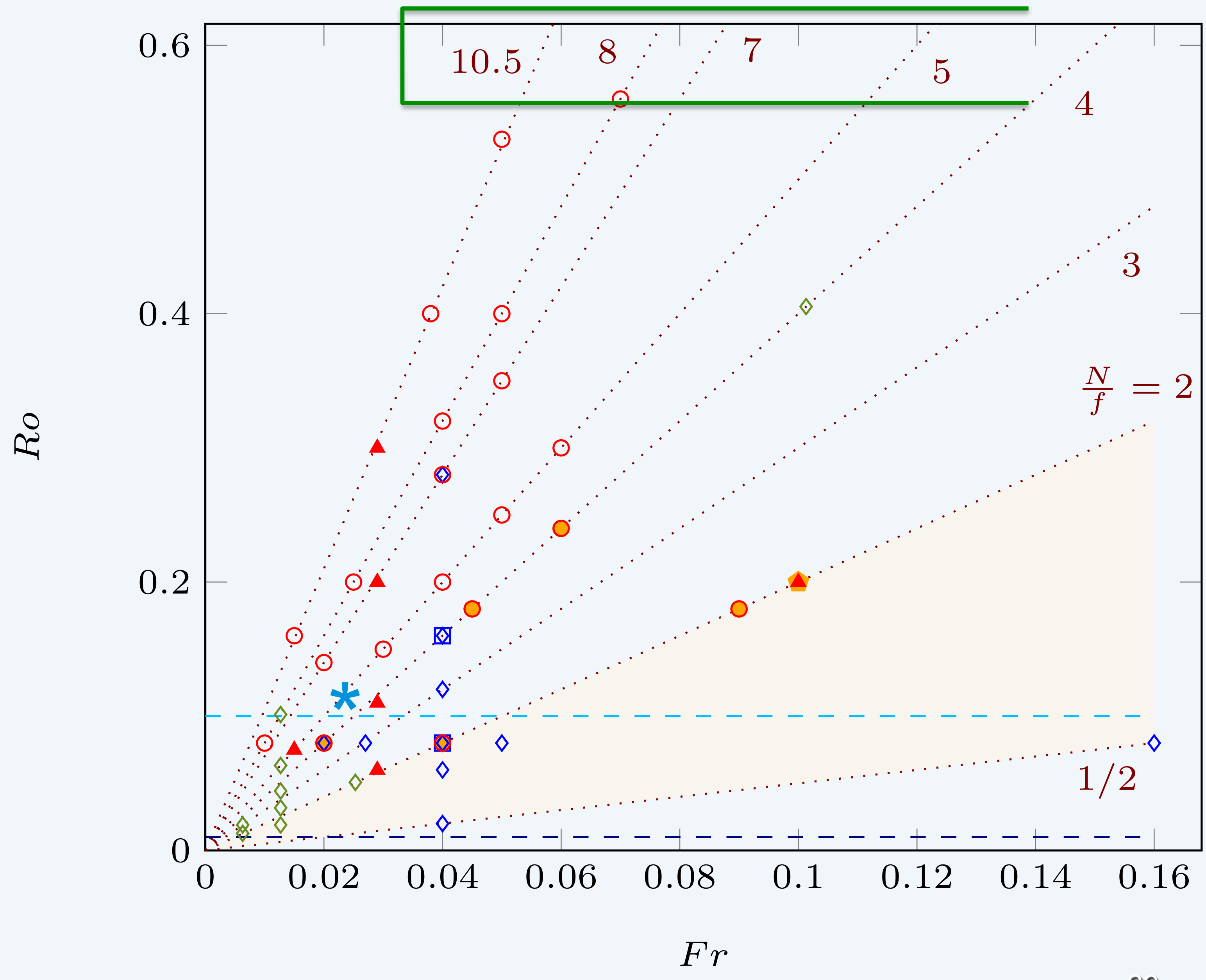
$\mathbf{R_\pi = \varepsilon_I/\varepsilon_D}$,

$E(k) \sim k^{-\alpha}$

Pouquet & Marino, PRL 2013

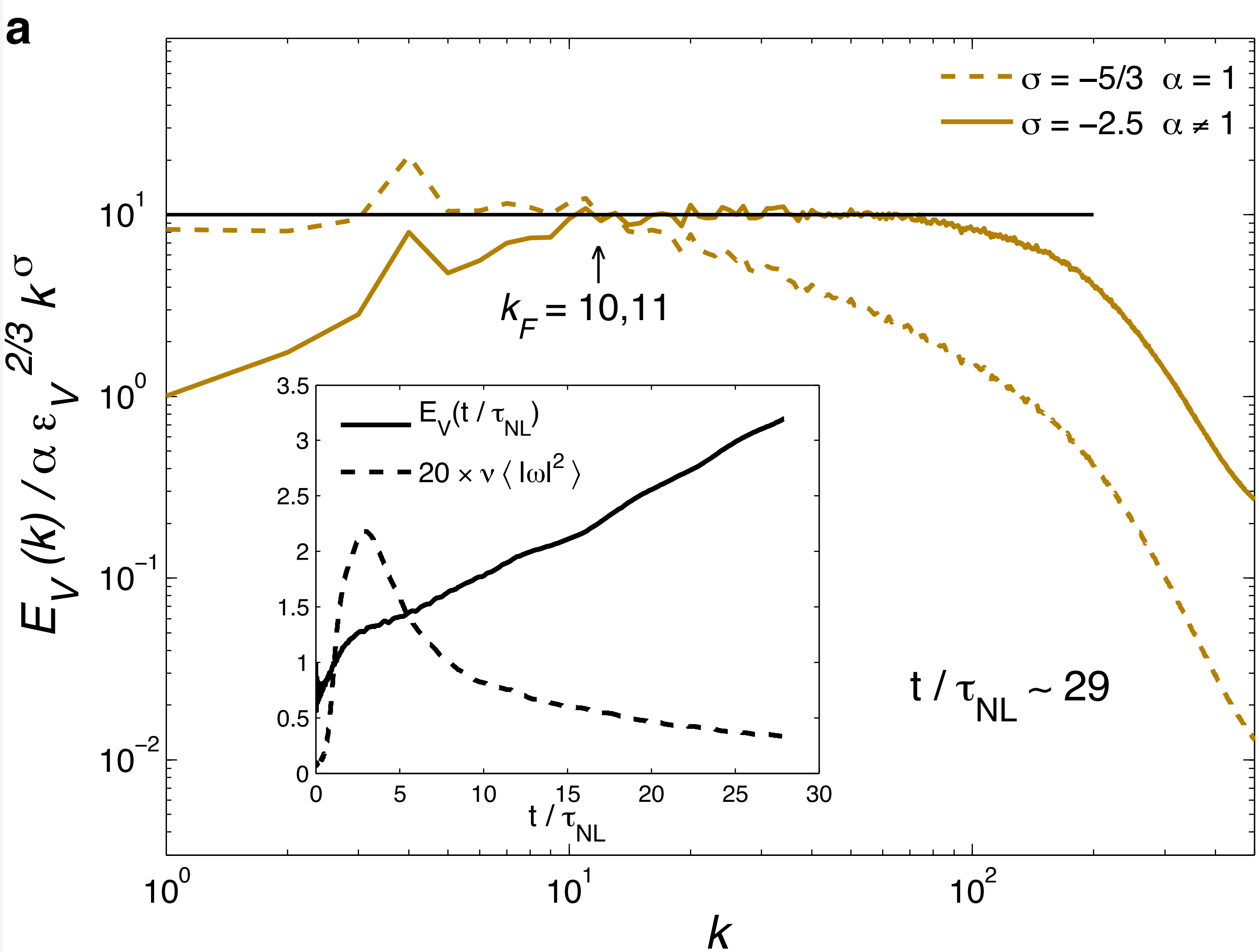


Horizont. slice of vertical velocity
R=8000
N/f=2
Fr=0.1
R_B=80
1536^3 res.
Forcing scale
Pouquet & Marino PRL 2013
1400
1200
1000
800
600
400
200
200
400
600
800
1000
1200
1400
0.6
0.4
0.2
0
−0.2
−0.4
−0.6

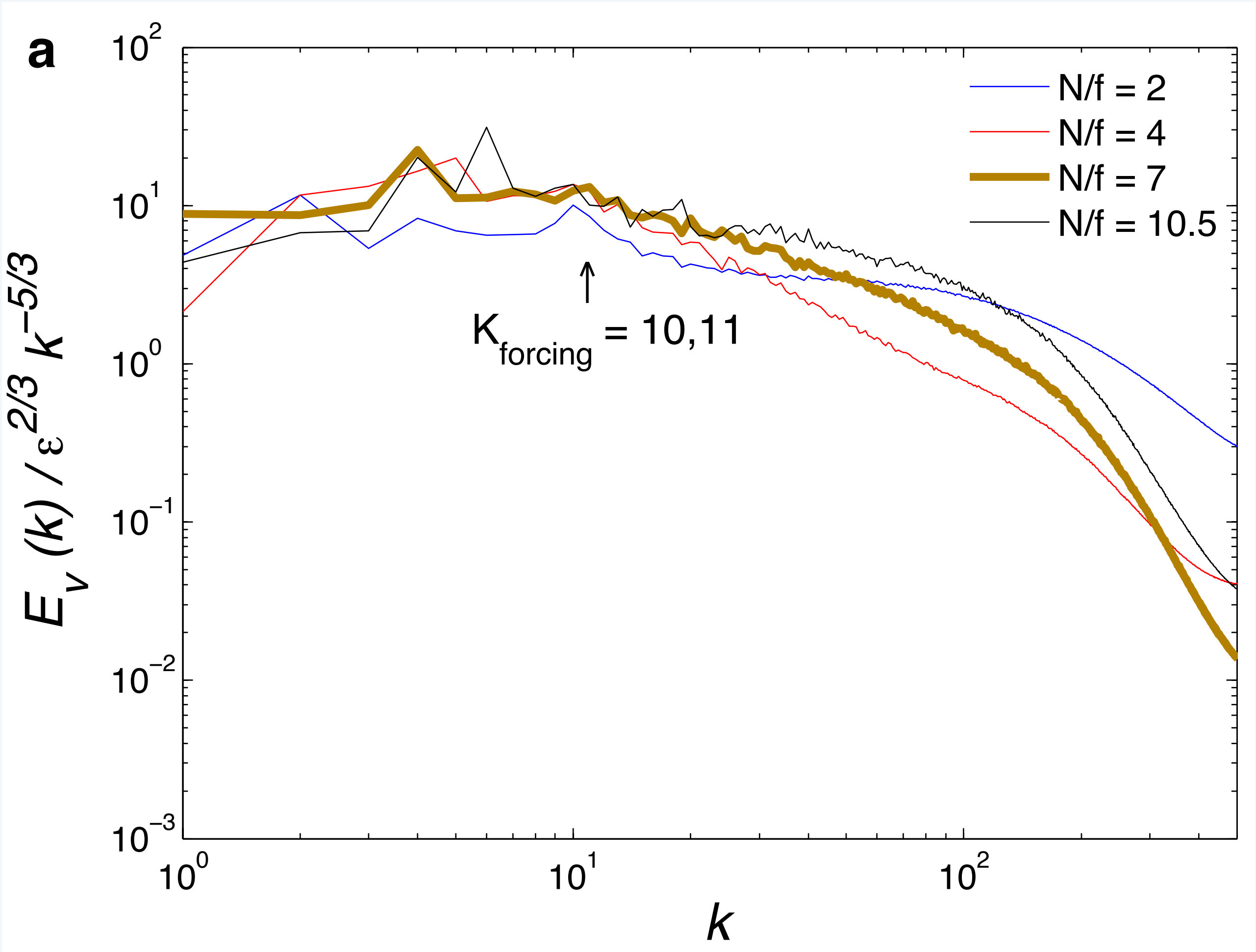


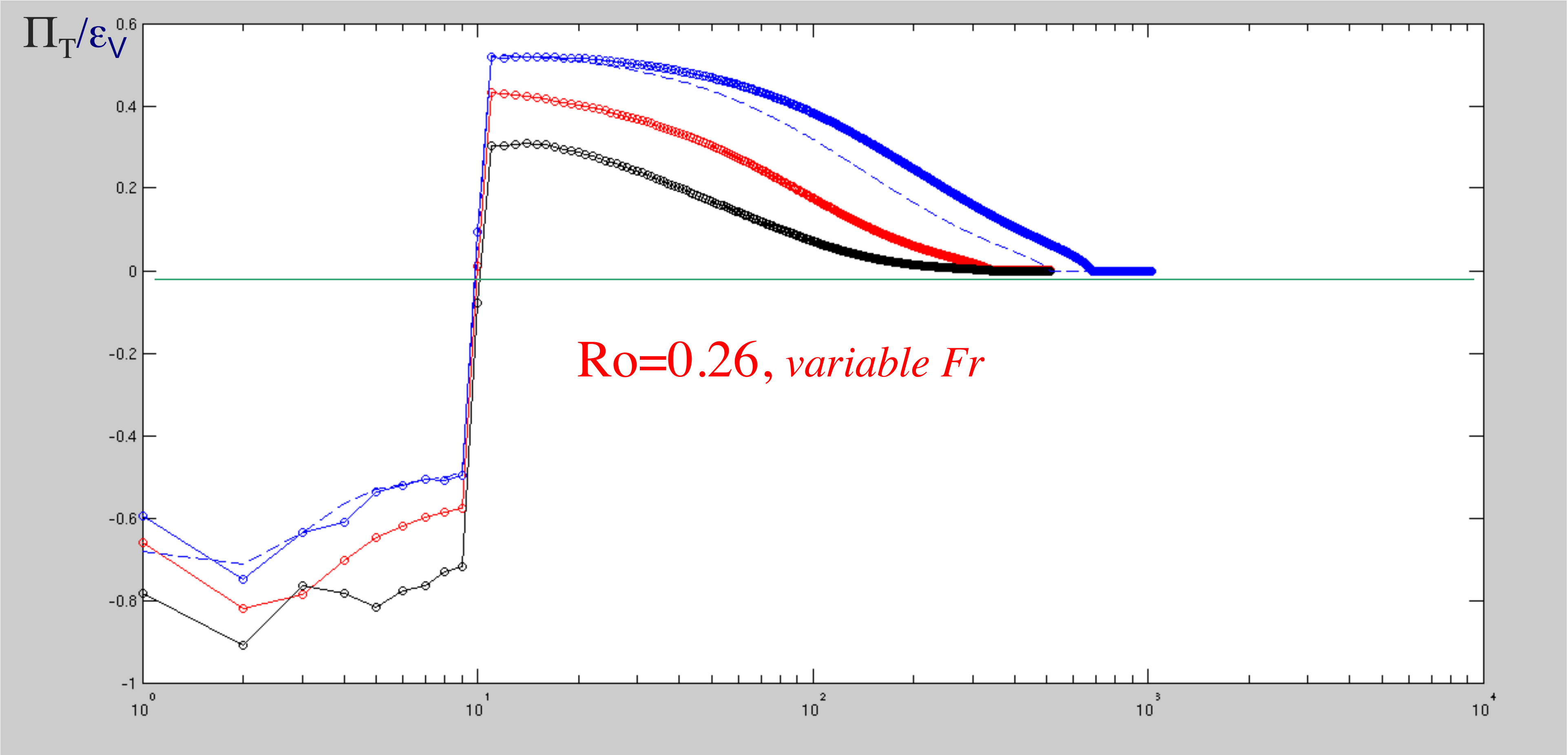
10.5
8
7
5
4
3
$\frac{N}{f} = 2$
1/2
0.6
0.4
0.2
0
Ro
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
Fr

TWO different compensations for the kinetic energy spectrum
N/f=7, Fr=0.047

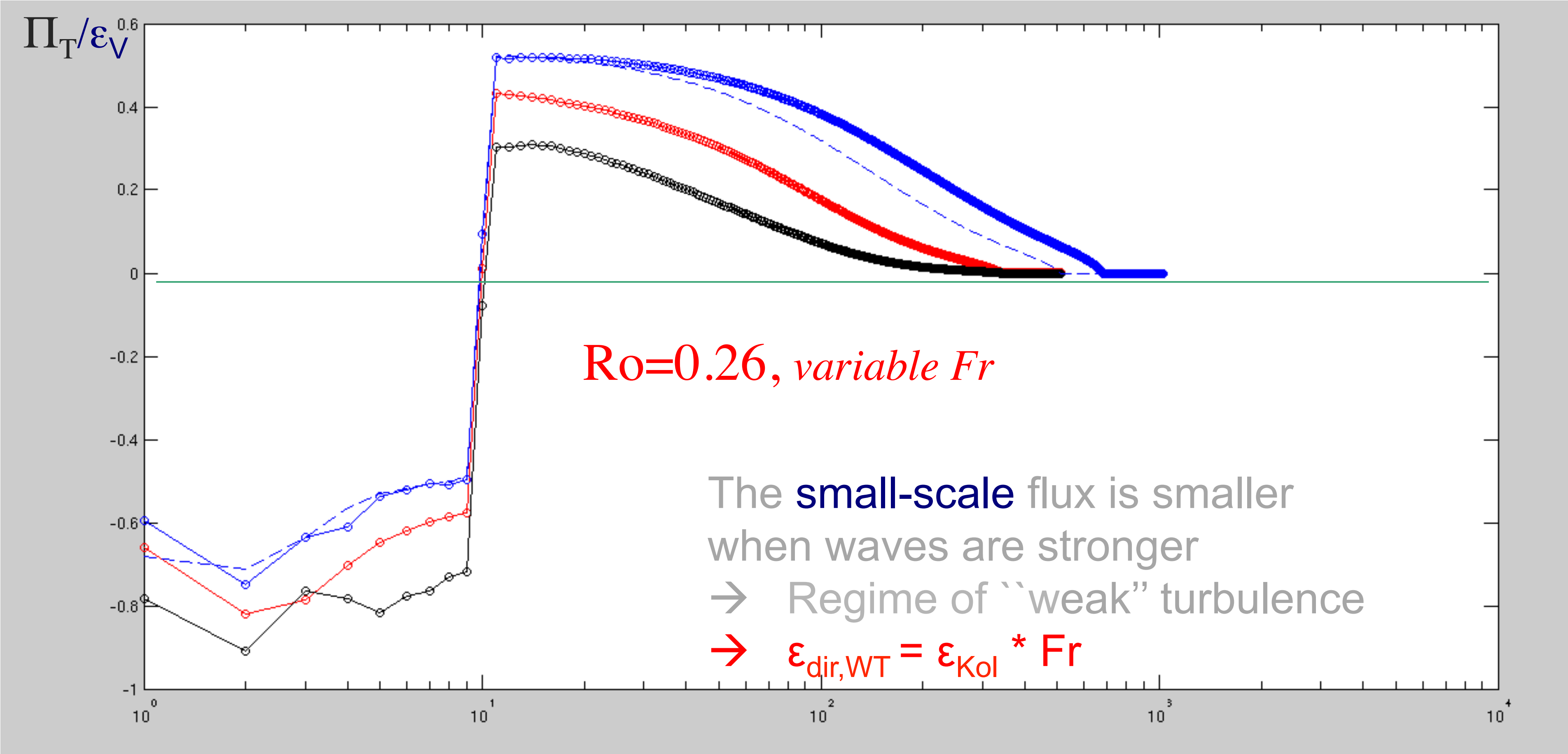


Kolmogorov constant for the inverse cascade

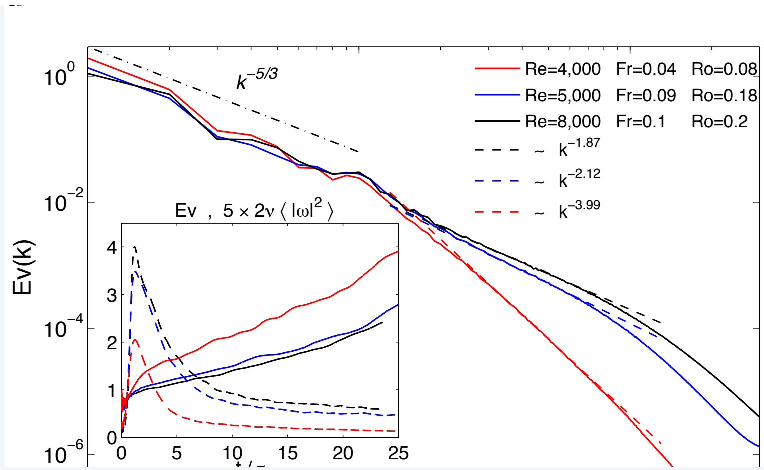




Π_T/ε_V
Ro=0.26, *variable Fr*
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
10^0
10^1
10^2
10^3
10^4



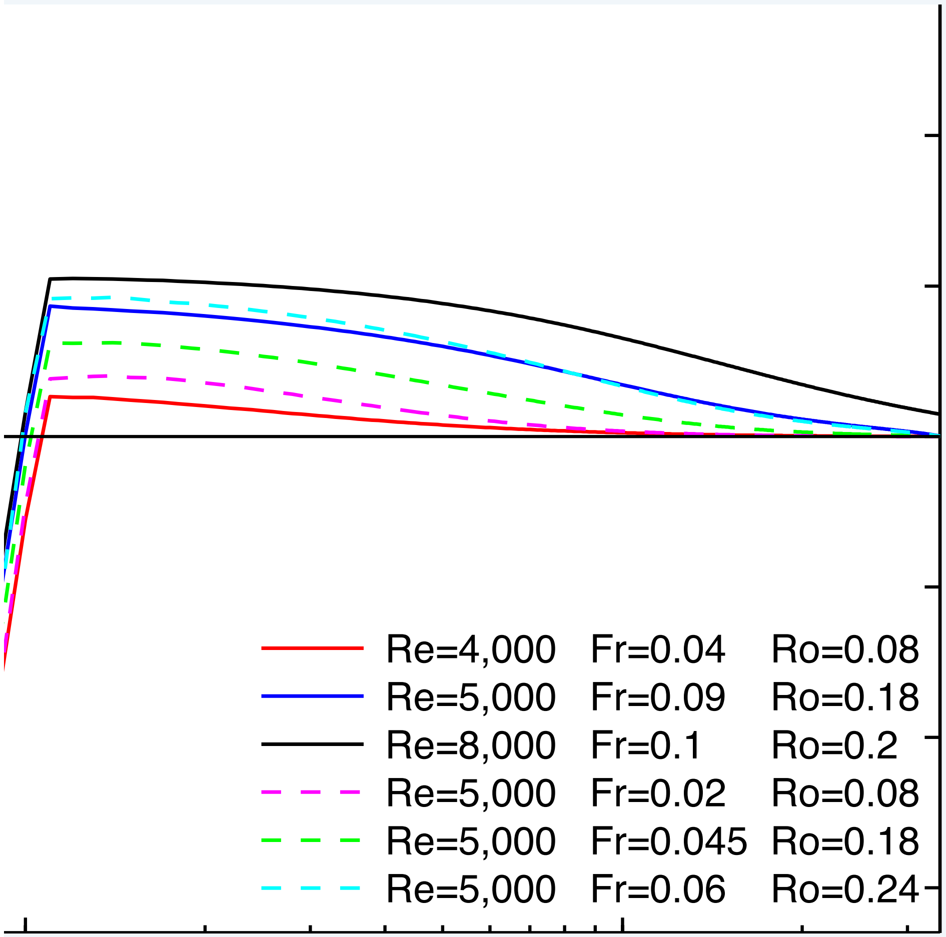
Π_T/ε_V
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
10^0
10^1
10^2
10^3
10^4
Ro=0.26, *variable Fr*
The small-scale flux is smaller
when waves are stronger
→ Regime of ``weak" turbulence
→ $\varepsilon_{dir,WT} = \varepsilon_{Kol} * Fr$



The **small-scale** flux is smaller when waves are stronger

→ Regime of ``weak'' turbulence

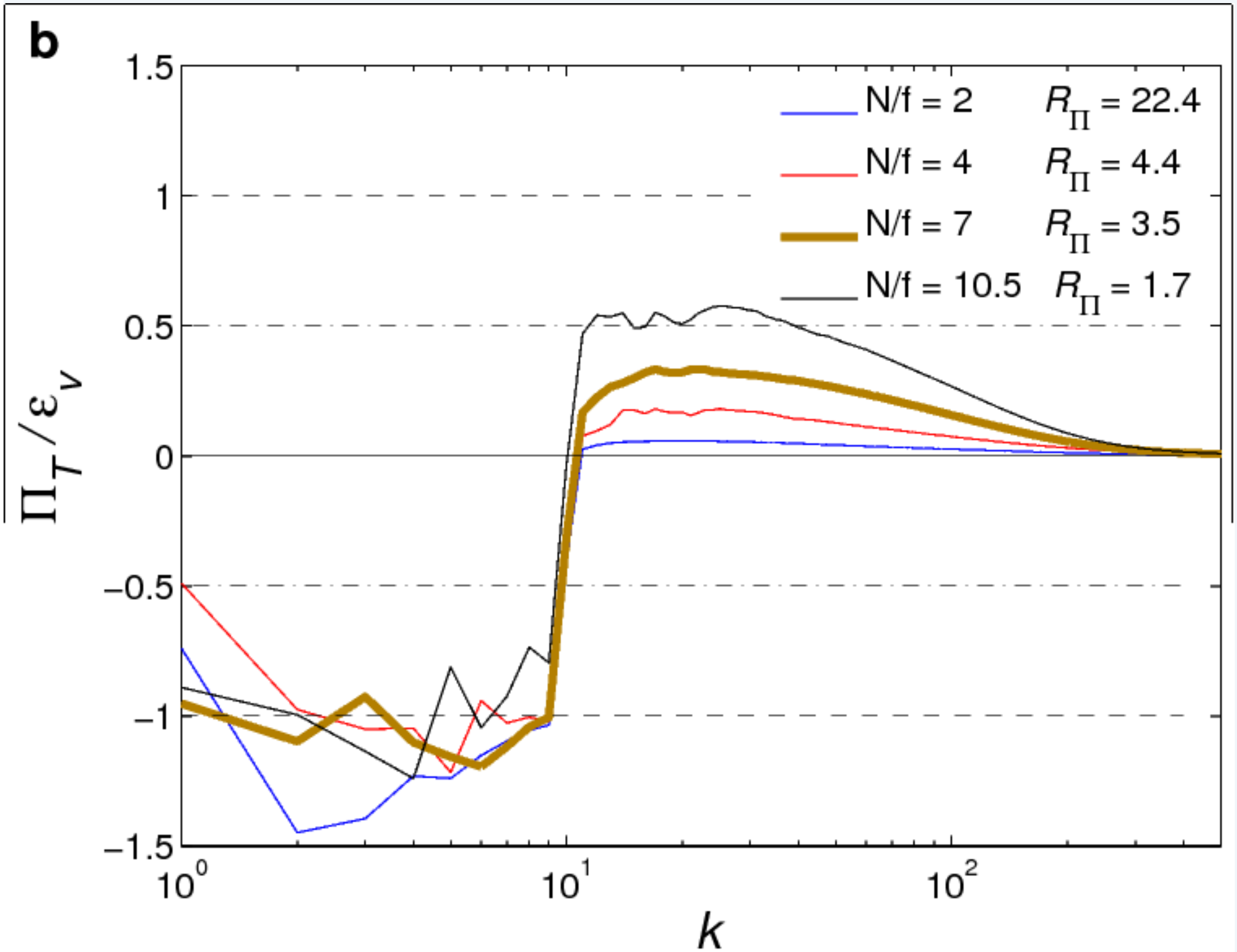
→ $\varepsilon_{dir,WT} = \varepsilon_{Kol} * Fr$



N/f=2 ____

N/f=4 - - - -

Energy flux



Fr=0.04, *variable Ro*

N/f=Ro/Fr, Ro=U/[Lf]

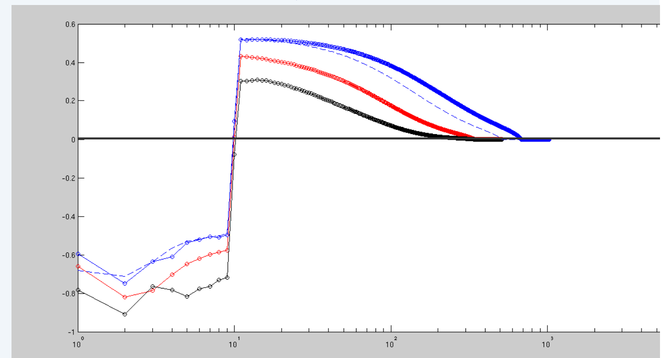
Ro~ 0.08
~ 0.16
~ 0.28
~ 0.45

*The stronger the rotation, the larger is R_Π, i.e. the larger is the cascade to large scales **relative** to that to small scales*

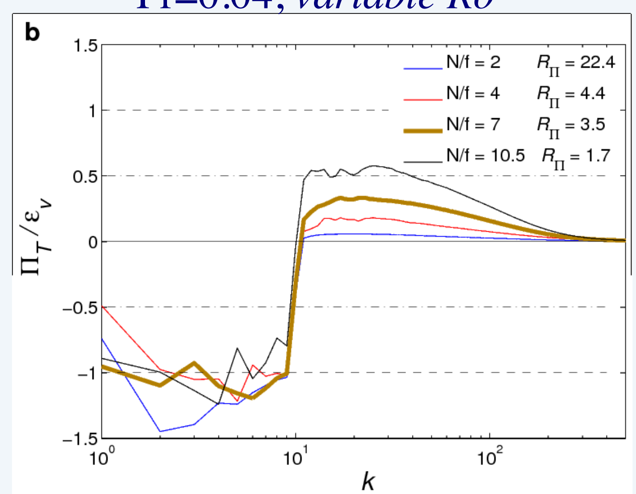
→ $\varepsilon_{LS} / \varepsilon_{ss} \sim [Fr * Ro]^{-1}$

$\sim \omega_{rms} [Nf]^{-1/2} Re^{-1}$

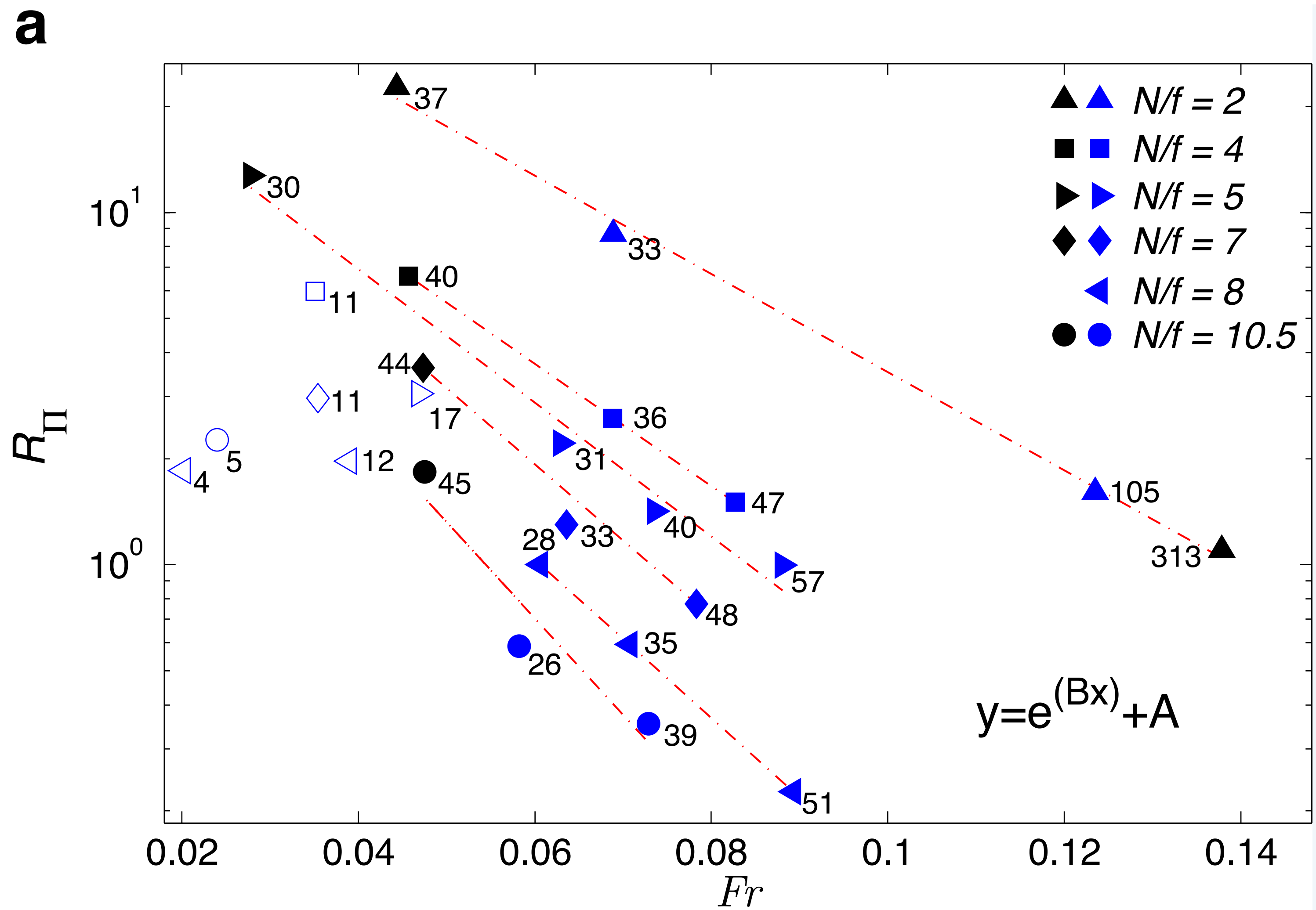
Ro=0.26, *variable Fr*



Fr=0.04, *variable Ro*

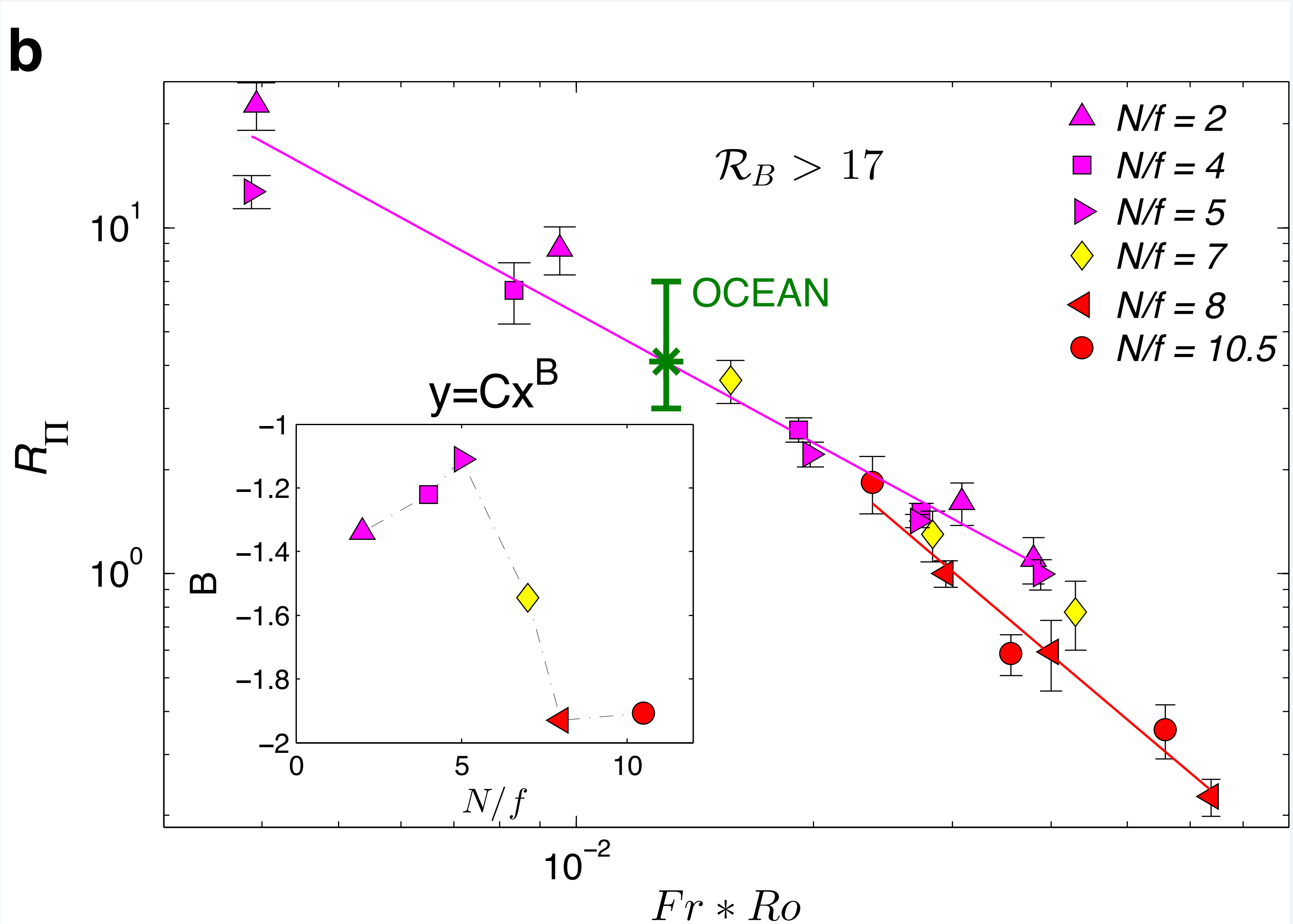


$R_\Pi = \varepsilon_{inv}/\varepsilon_{dir}$

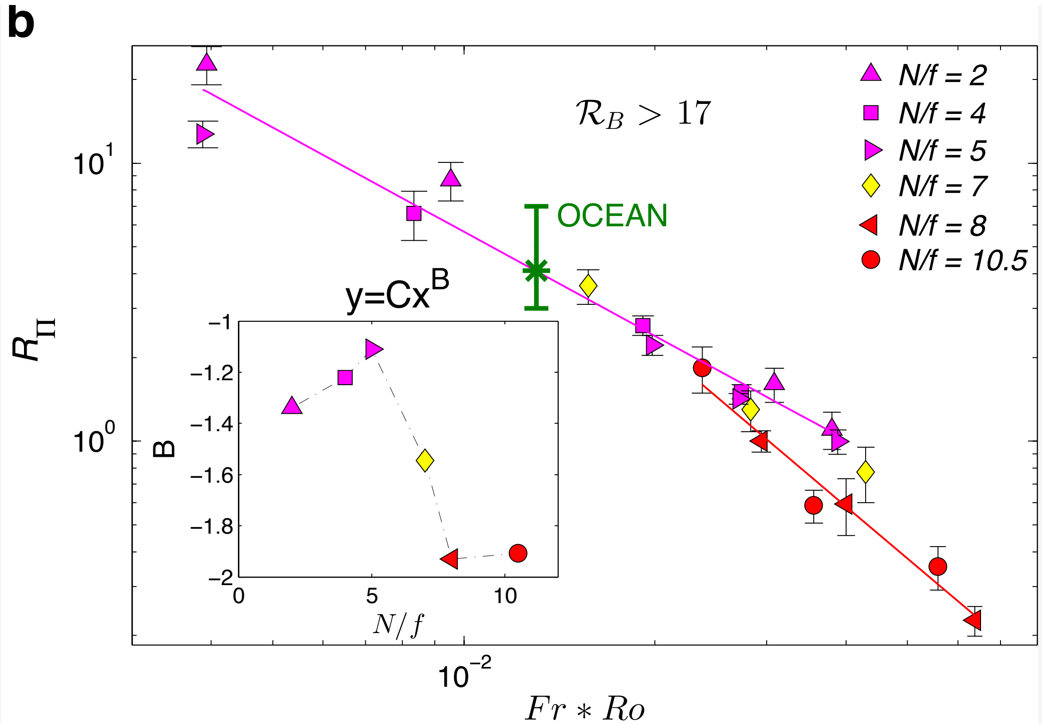


* Point labeled with values of $R_B = Re\, Fr^2$

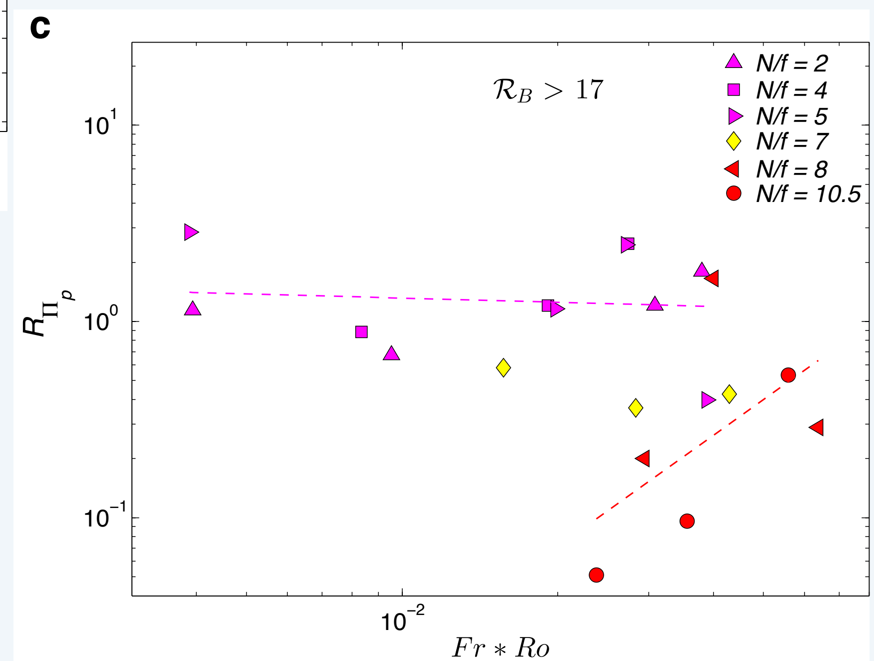
$R_\Pi = \varepsilon_{inv}/\varepsilon_{dir}$



R_B = Re Fr^2

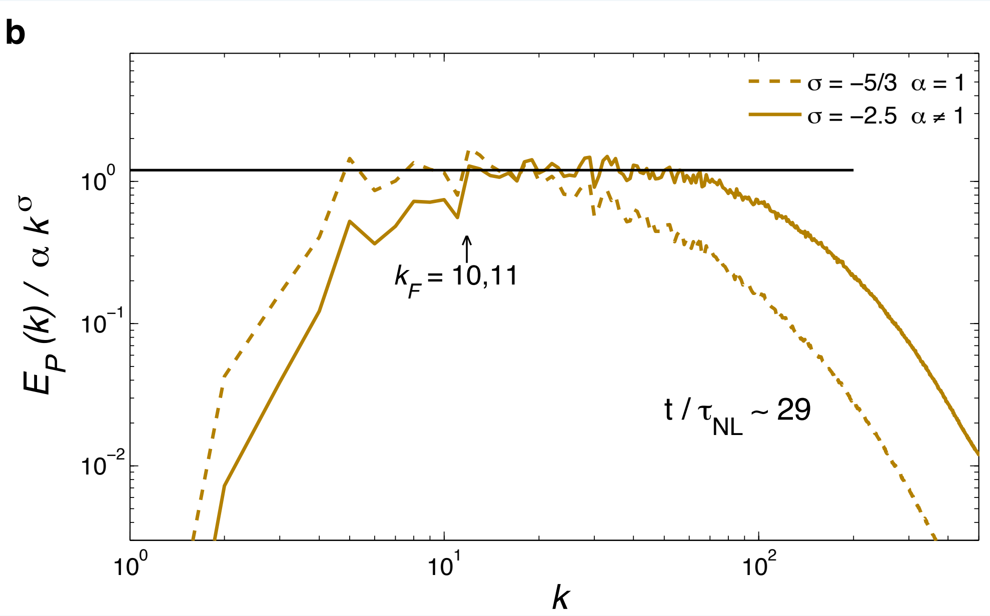


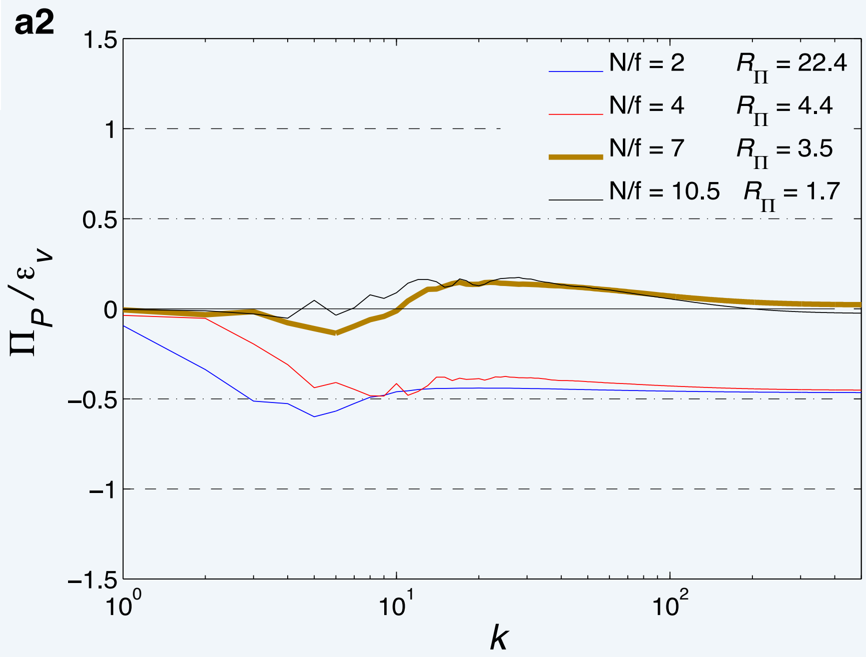
$R_\Pi = \varepsilon_{inv}/\varepsilon_{dir}$



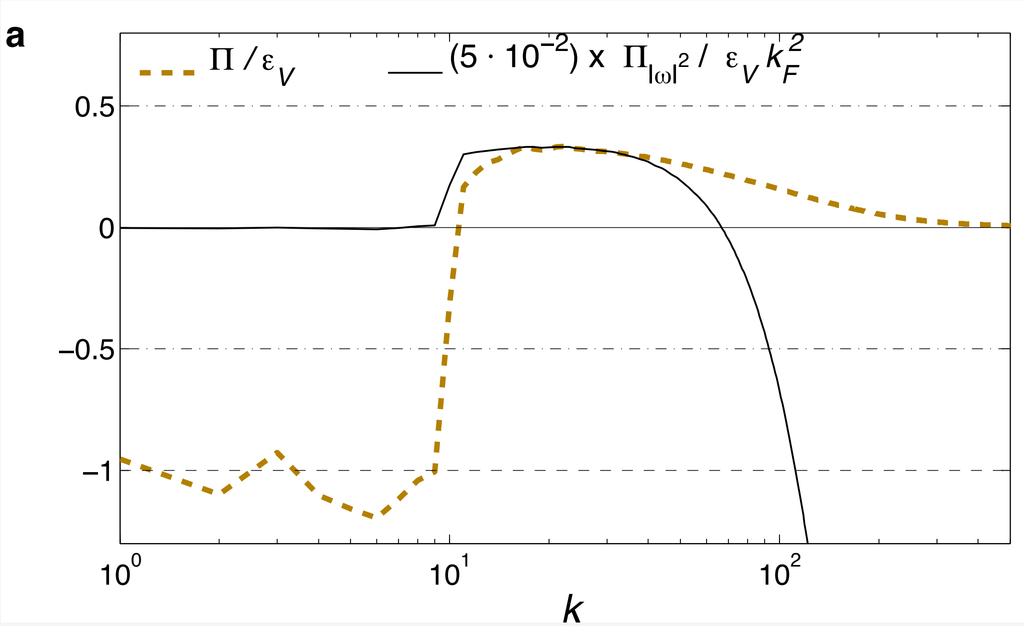
Is N/f ~ 7 special? Why a transition there?

Inverse cascade of potential energy?





Is there an enstrophy cascade?



Summary, *future work and open questions*

Scaling with $[Fr^*Ro]^{-1}$ of the flux ratio of the bi-directional cascade

- *Anisotropic analysis & normal modes decomposition*
- *Role of helicity? Role of conservation of potential vorticity?*
- *Cascade of enstrophy? Of potential energy?*

- *Long-time accumulation at k=1, & large-scale friction?*
- *Different forcing, e.g. two-dimensional or balanced?*
- *Criticality?*

- *Lagrangian particles, mixing and passive scalar in a dual cascade*

Different regimes: *What are the characteristic break-up scales for energy partition, and how do such scales vary with parameters?*

- *Modeling with anisotropic eddy viscosity (>0, <0)?*

- Thank you for your attention