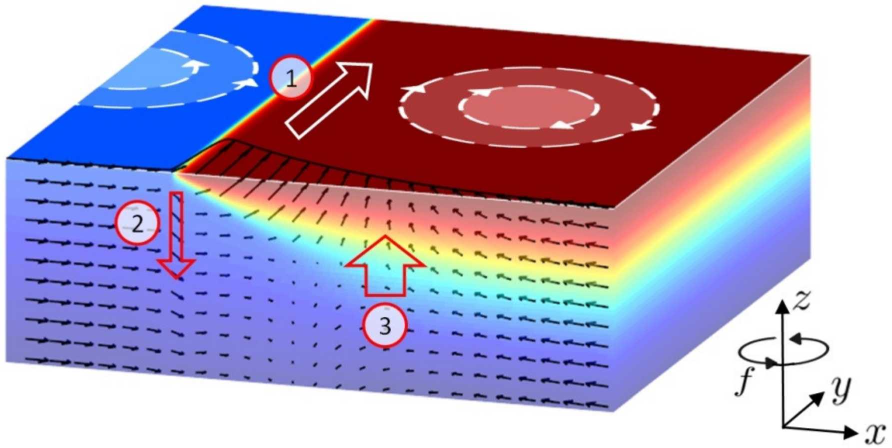
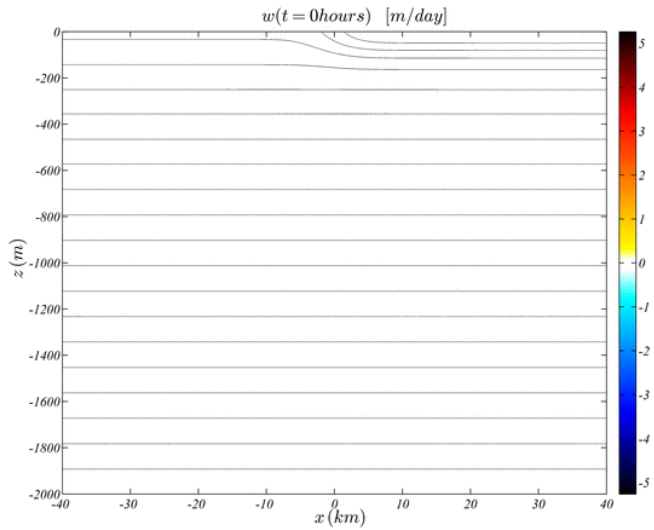
“The Frontal Mountain Effect”

Spontaneous wave generation at strained density fronts





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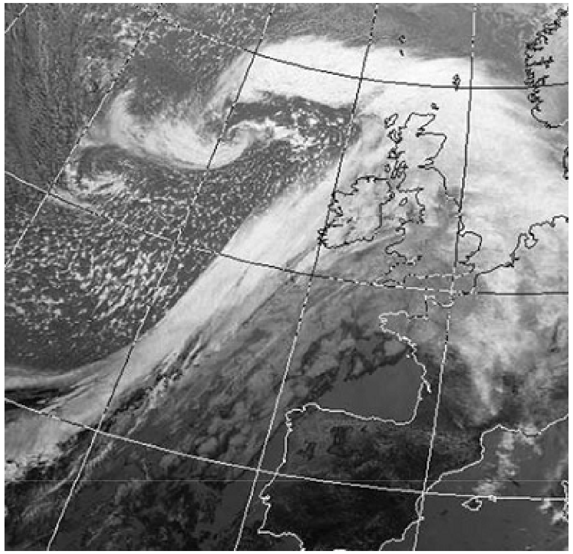
Department of Applied Mathematics and Theoretical Physics

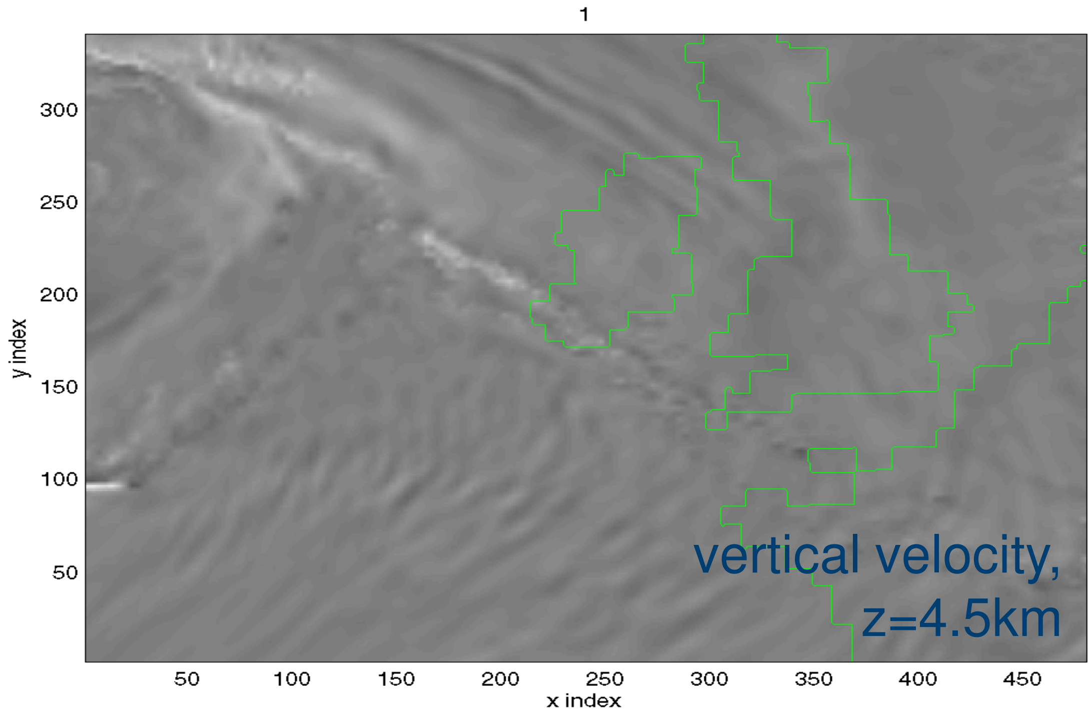
Les Houches, France, March 2, 2015

Motivation



Cold front wave generation event:

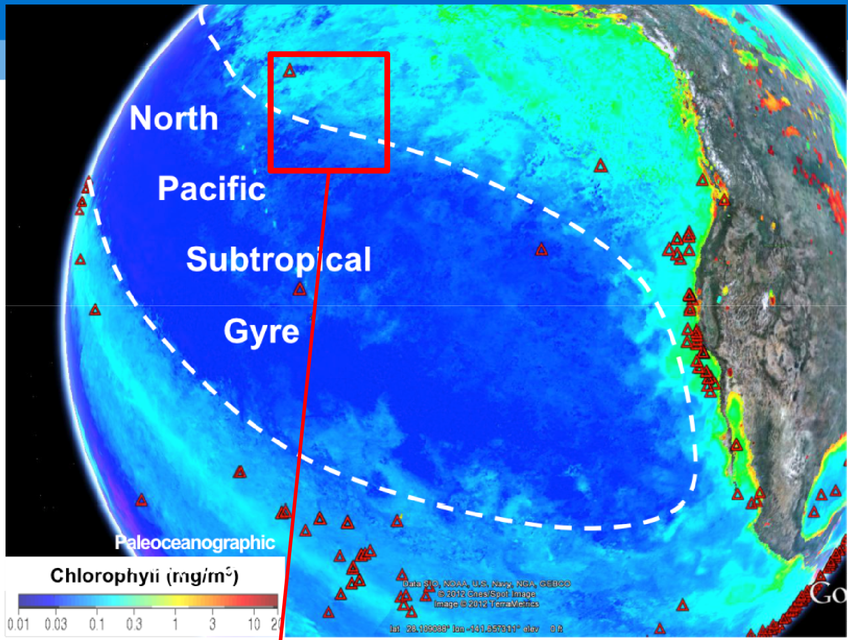
Knippertz et al. (2010)



Movie: J. Methven & B. Harvey, U. Reading

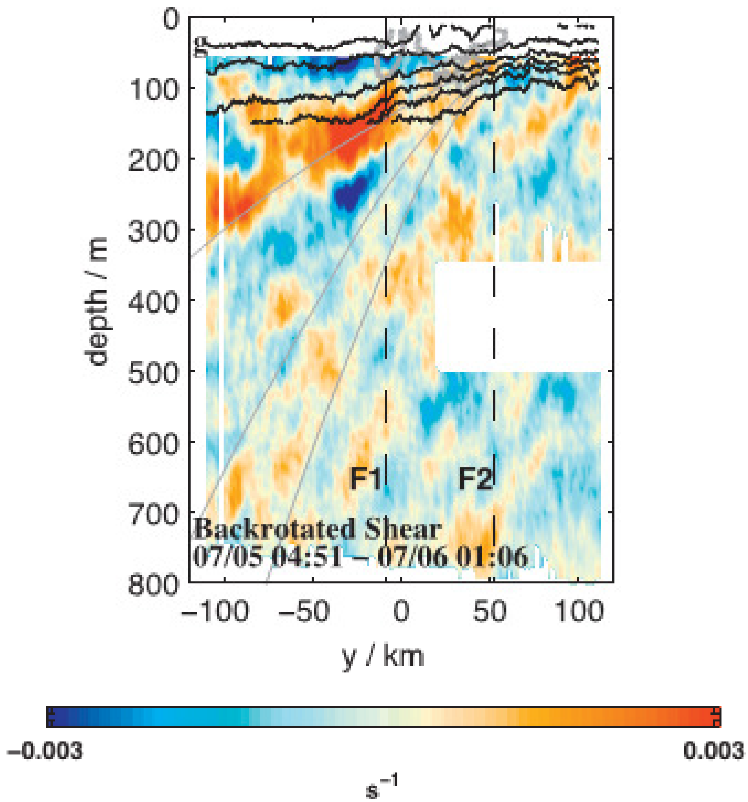
Motivation

survey region

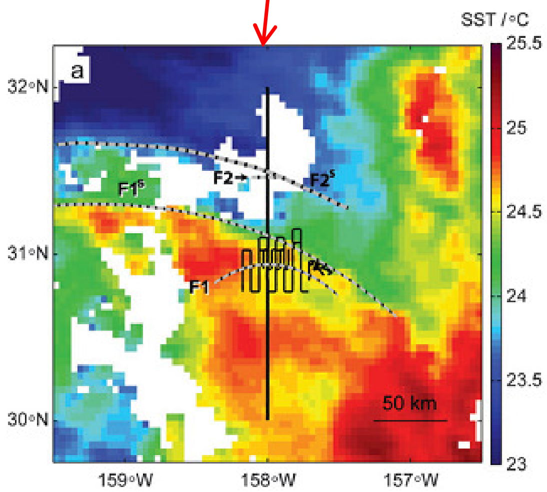


Ocean front wave generation event: Alford et al. (2013)

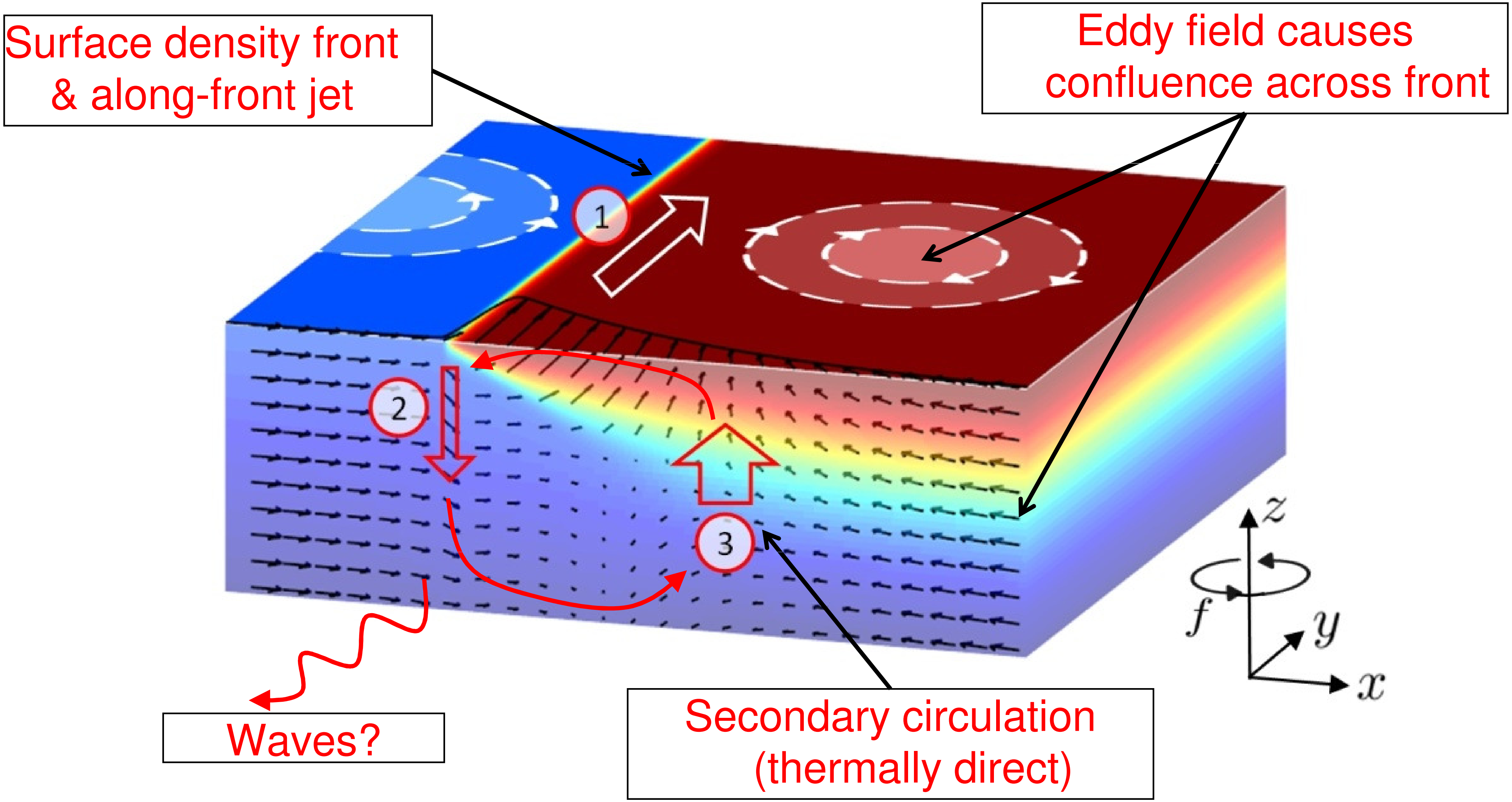
cross-frontal shear



Sea Surface Temperature (SST)



A model of strain-forced frontogenesis (e.g. Hoskins and Bretherton, 1972)

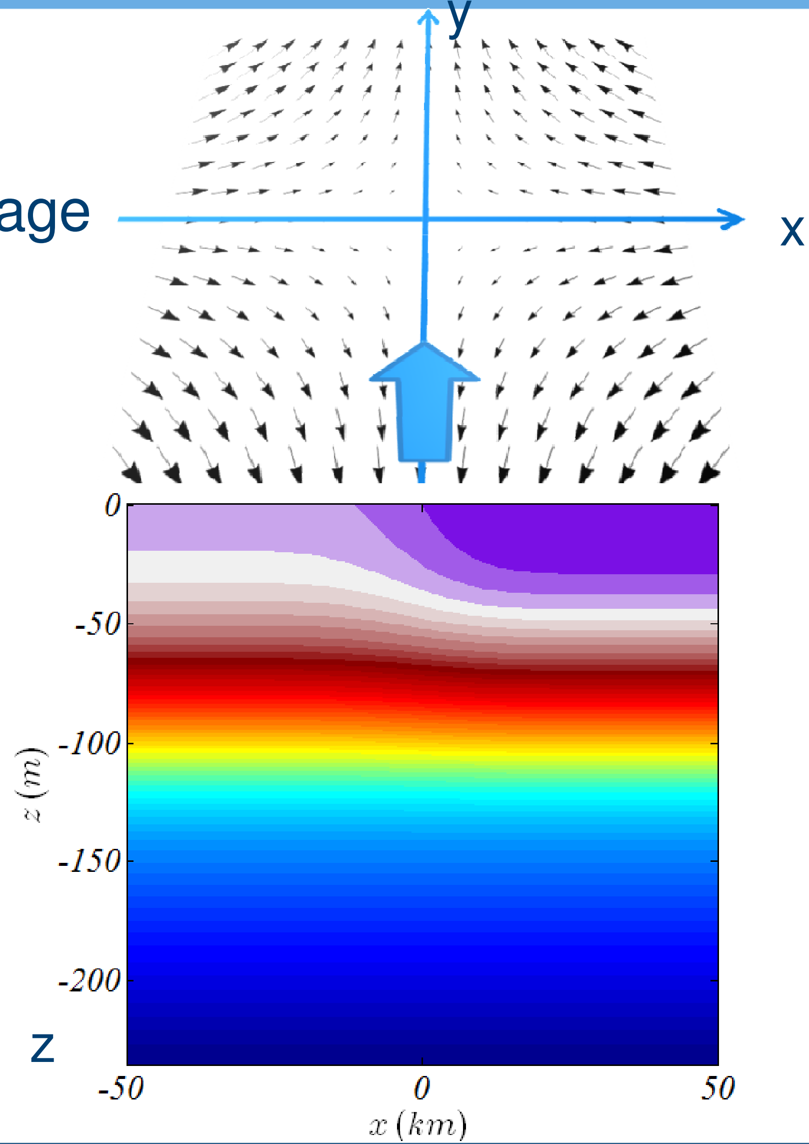


A quasi-2D model, following HB72

- Assume that:
  - Two-dimensional: front infinitely long into page
    - No *y* derivatives
  - Background strain flow uniform in z
    - simplest possible form

$$U_0 = -\alpha x \text{ and } V_0 = \alpha y$$

  - Uniformly stratified ambient (**)
  - Hydrostatic (**)



Shakespeare and Taylor 2015, submitted.

Frontal waves - simple quasi-2D linear model

2D linear equations:

$$\bar{D}u = fv \underline{+ \alpha u} - f v_g,$$
$$\bar{D}v = - fu \underline{- \alpha v,}$$
$$\frac{\partial v_g}{\partial z} = \frac{1}{f}\frac{\partial b}{\partial x},$$
$$\bar{D}b = -N^2 w$$

Linearised material derivative:

$$\bar{D} = \partial_t - \underline{\alpha x \, \partial_x}$$

(background advection only)

PV conservation: $\bar{D}q = 0$

$$q = f\frac{\partial b}{\partial z} + N^2 \frac{\partial v}{\partial x}$$

- Valid for:

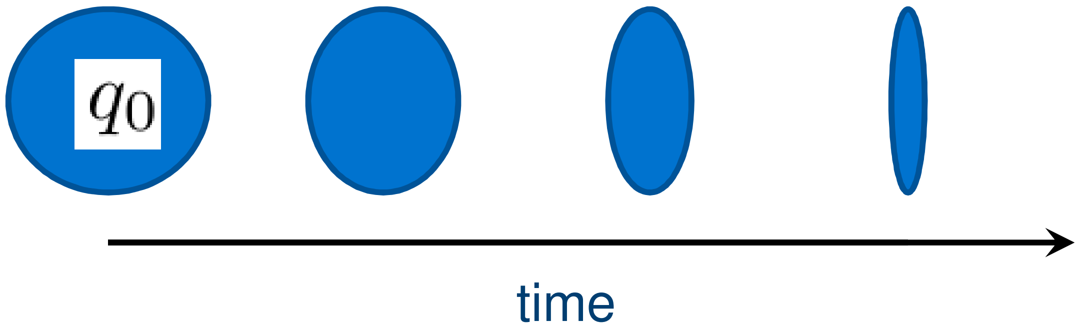
$$Ro = \frac{1}{f}\frac{\partial v}{\partial x} = \frac{\Delta b \, H}{f^2 L^2} \ll 1$$

Shakespeare and Taylor 2015, submitted.

Frontal waves - simple 2.5D linear model

- Consider the evolution of the PV:

$$\bar{D}q = (\partial_t - \alpha x\, \partial_x)q = 0 \implies q = q_0(xe^{\alpha t},\ z)$$



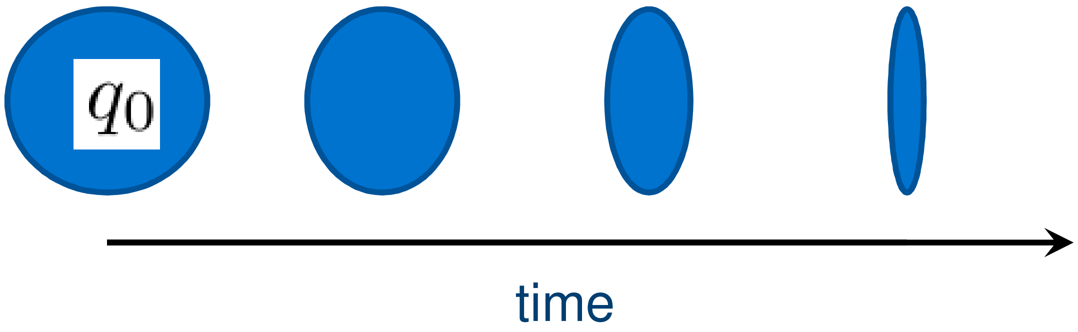
The strain squeezes an initial PV anomaly with time

Frontal waves - simple 2.5D linear model

$$B = b' + b_0(x,z) + N^2 z$$

- Consider the evolution of the PV:

$$\bar{D}q = (\partial_t - \alpha x\,\partial_x)q = 0 \implies q = q_0(xe^{\alpha t},\, z)$$



The strain squeezes an initial PV anomaly with time

- The associated frontal anomaly also collapses = “frontogenesis”

$$b_0(xe^{\alpha t},\, z) = \frac{1}{f}\int q_0(xe^{\alpha t},\, z)\,dz$$

- What is the response of the flow to this PV/frontal collapse?

Shakespeare and Taylor 2015, submitted.

Frontal waves - simple 2.5D linear model

$$B = b' + b_0(xe^{\alpha t},\, z) + N^2 z$$

- The perturbation buoyancy b', is forced by the collapse of the PV anomaly:

$$\left[\underbrace{\left(\bar{D}^2 - 2\alpha\bar{D}\right.}_{\substack{\text{ageostrophic:}\\ \text{cross-front}\\ \text{accelerations}}} \underbrace{\left. + f^2\right)\frac{1}{N^2}\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right]}_{\substack{\text{geostrophic:}\\ \text{QG frontogenesis}\\ \text{Williams \& Plotkin '78}}} b' = \underbrace{-\frac{\partial^2}{\partial x^2} b_0\left(x\, e^{\alpha t},\, z\right)}_{\substack{\text{forcing:}\\ \text{collapse of the}\\ \text{frontal anomaly}}}$$

- Time-dependent forcing => no steady state solution!

Shakespeare and Taylor 2015, submitted.

Frontal vs. Lee waves: uniform interior PV

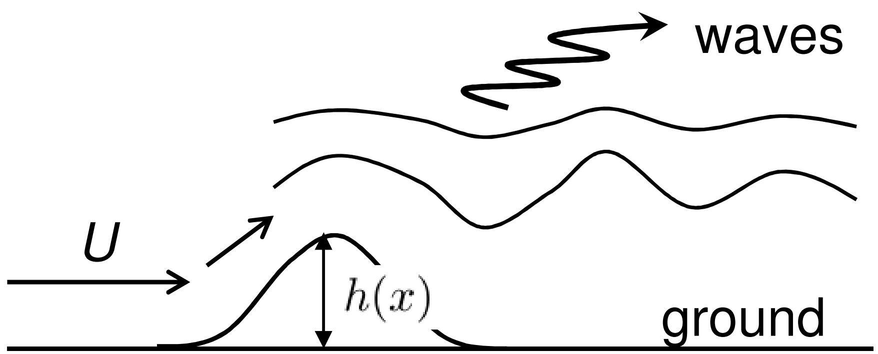
$b_0 = b_0(xe^{\alpha t})$

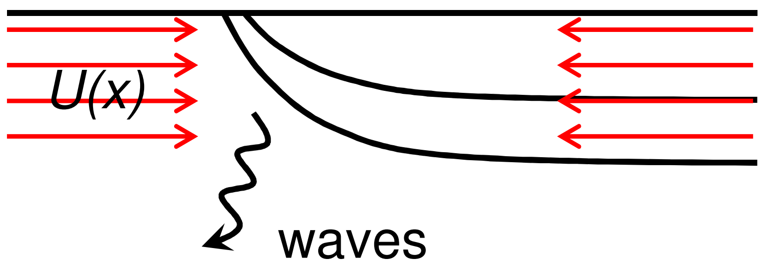
- The governing equation becomes, subject to... boundary condition

$$\left[\left(\underline{\bar{D}^2 - 2\alpha\bar{D}} + f^2\right)\frac{1}{N^2}\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right] b = 0 \qquad b(x,0) = b_0(xe^{\alpha t})$$

- c.f. for lee waves (e.g. Queney 1947, Pierrehumbert 1984)

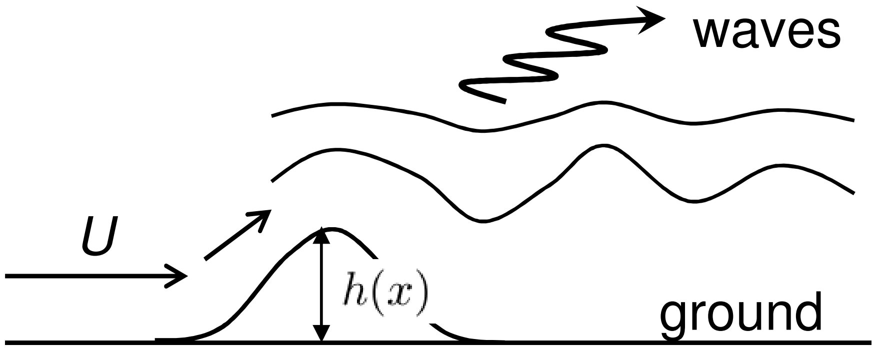
$$\left[\left(\underline{\bar{D}}^2 + f^2\right)\frac{1}{N^2}\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right] b = 0 \qquad b(x,0) = -N^2\,h(x)$$



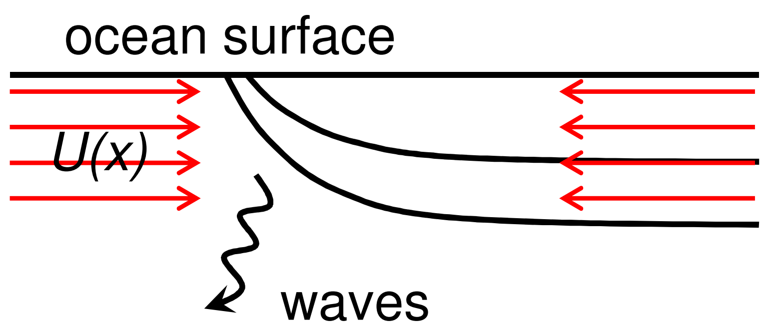


Frontal vs. Lee waves: uniform interior PV

External obstacle: ridge



Internal obstacle: front



Wave generation occurs in each case due to the acceleration of the background flow around the obstacle (ridge/front), if it is sufficiently sharp (D^2 large)

In other words: density fronts in a strain flow are like mountains in a uniform background flow.

aka: *The Frontal Mountain Effect*

For what strain do wave appear?

$$\delta = \frac{\alpha}{f}$$

- Consider a buoyancy anomaly

$$\frac{\partial b_0}{\partial x} = \exp\left[-\left(\epsilon_F \frac{x}{L_R}\right)^2 - \left(\frac{z}{H}\right)^2\right]$$

- Forcing slope

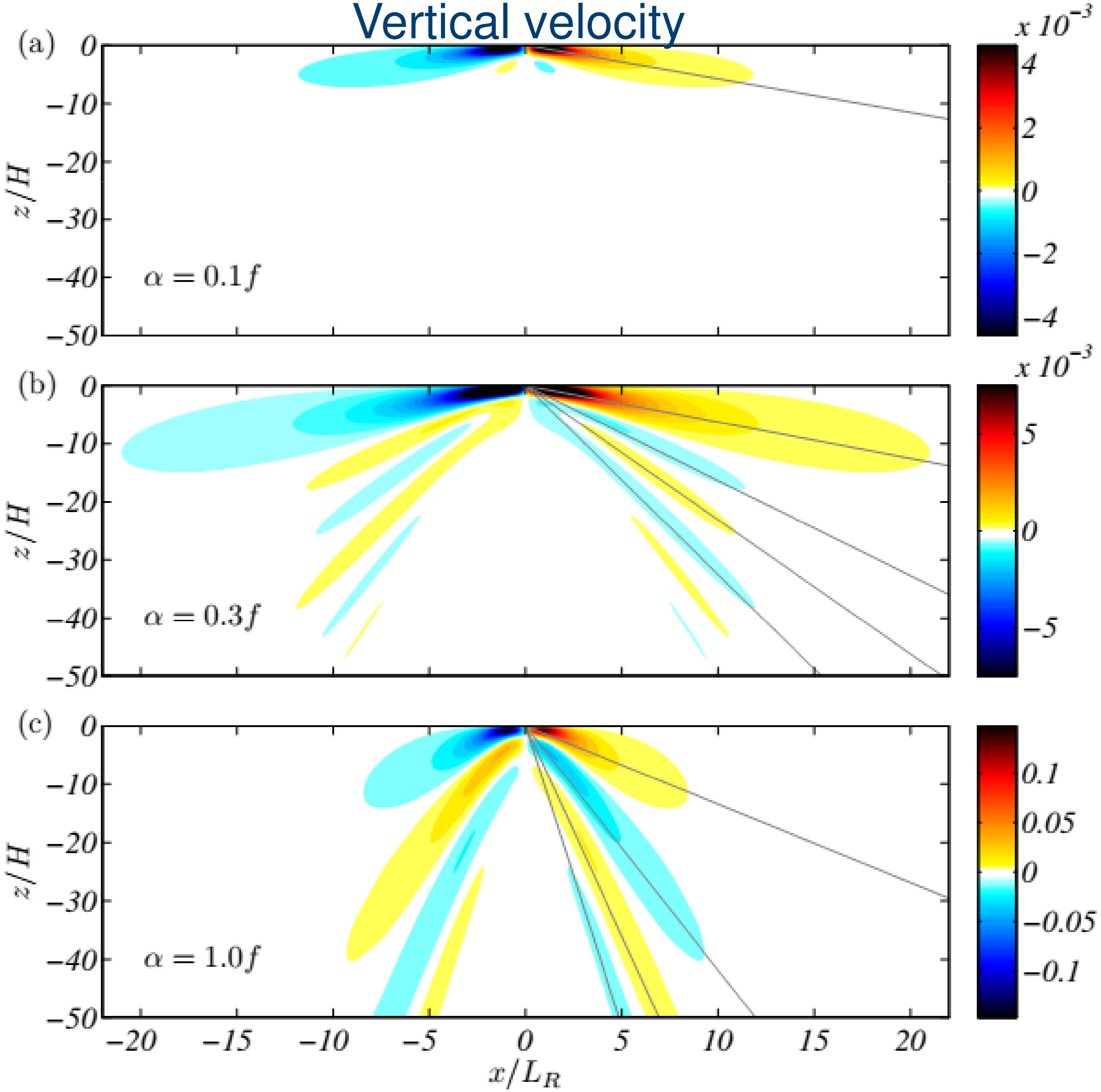
$$\epsilon_F = \frac{L_R}{L} = 1/2$$

- Rossby radius

$$L_R = \frac{NH}{f}$$

<u>For wave generation:</u>
Strain >> 0.1f

Vertical velocity



For what frontal scale do waves appear?

$$\epsilon_F = \frac{L_R}{L}$$

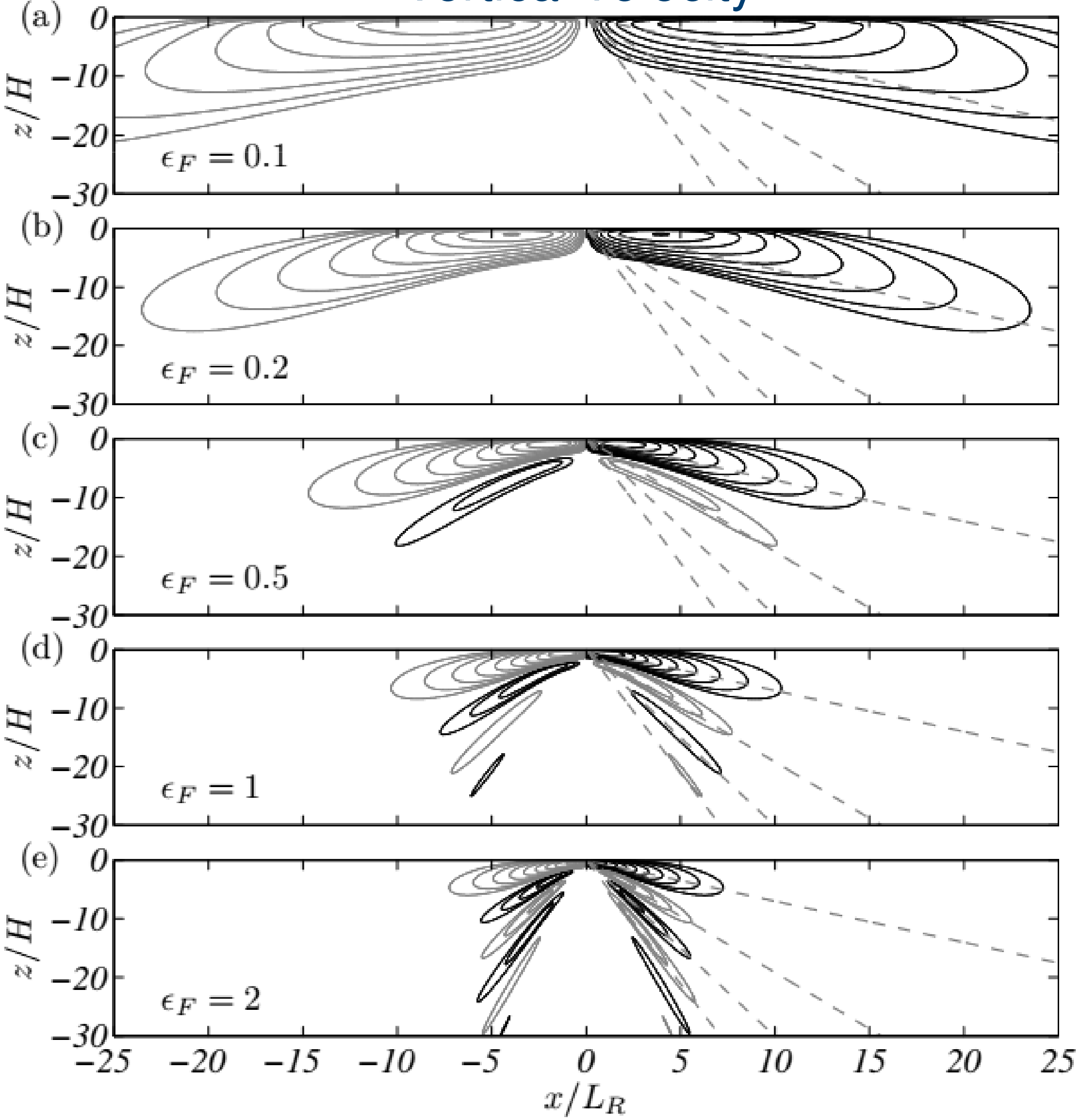
$\delta = 0.4$

- Frontal scale contracts with time
- Burger number increases

$$\epsilon_F(t) = \epsilon_{F,0}\, e^{\alpha\, t}$$

<u>For wave generation:</u>
frontal scale must be order of Rossby radius or smaller

Vertical velocity



Numerical simulation: MITgcm

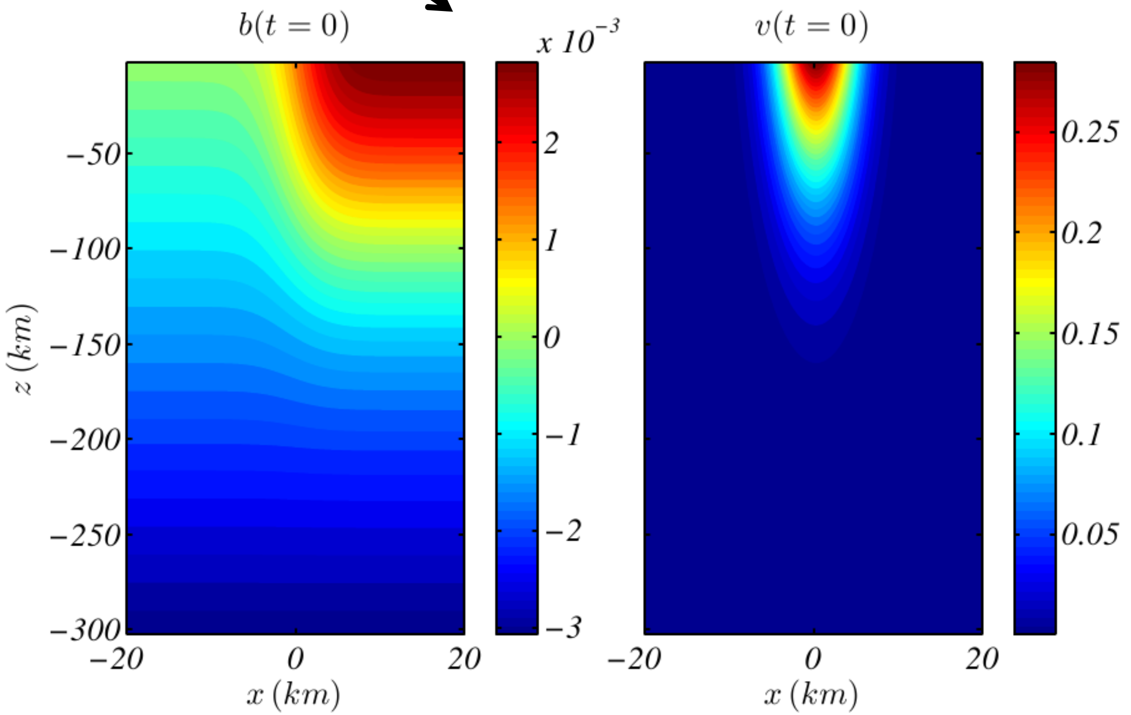
- Let's compare with some fully non-linear simulations (MITgcm)
- Initial condition of geostrophic balance
- Gradually turn on strain with time:

$$\alpha(t) = \alpha_0 \left(1 - e^{-(t/\tau)^2}\right)$$

- Here we choose: $\tau = 1\,\mathrm{day}$

$$\alpha_0 = 0.4f$$

$$Ro = \frac{\Delta b\, H}{f^2\, L^2} = \frac{(5 \times 10^{-3})(100)}{(10^{-4})^2(10^3)^2} = 30$$



Numerical simulation: MITgcm

- Add some horizontal diffusion/viscosity to prevent the collapse of the front beneath the grid scale

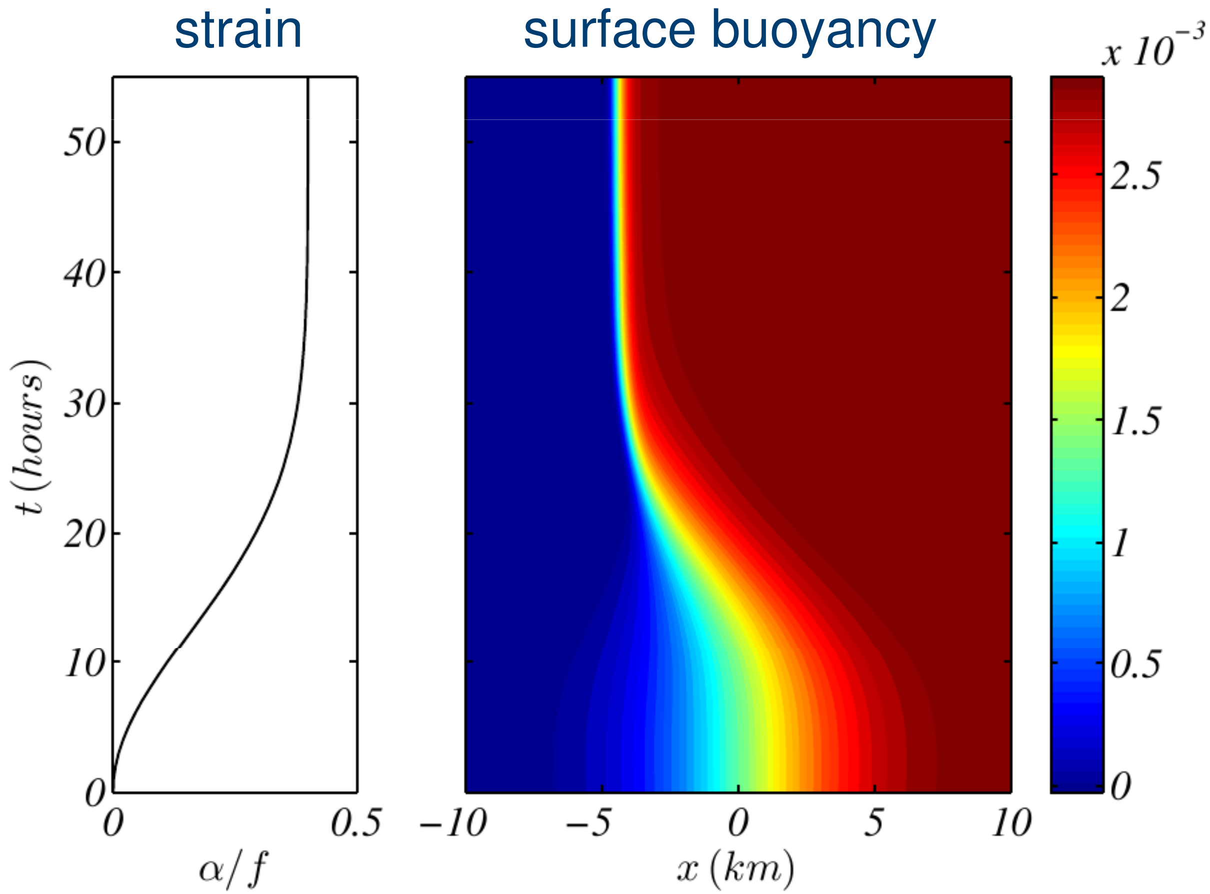
$$\kappa_h = \nu_h = 10\, m^2\, s^{-1}$$

- Front ultimately reaches a steady state:

$$-\alpha x \frac{\partial b}{\partial x} = \kappa_h \frac{\partial^2 b}{\partial x^2}$$

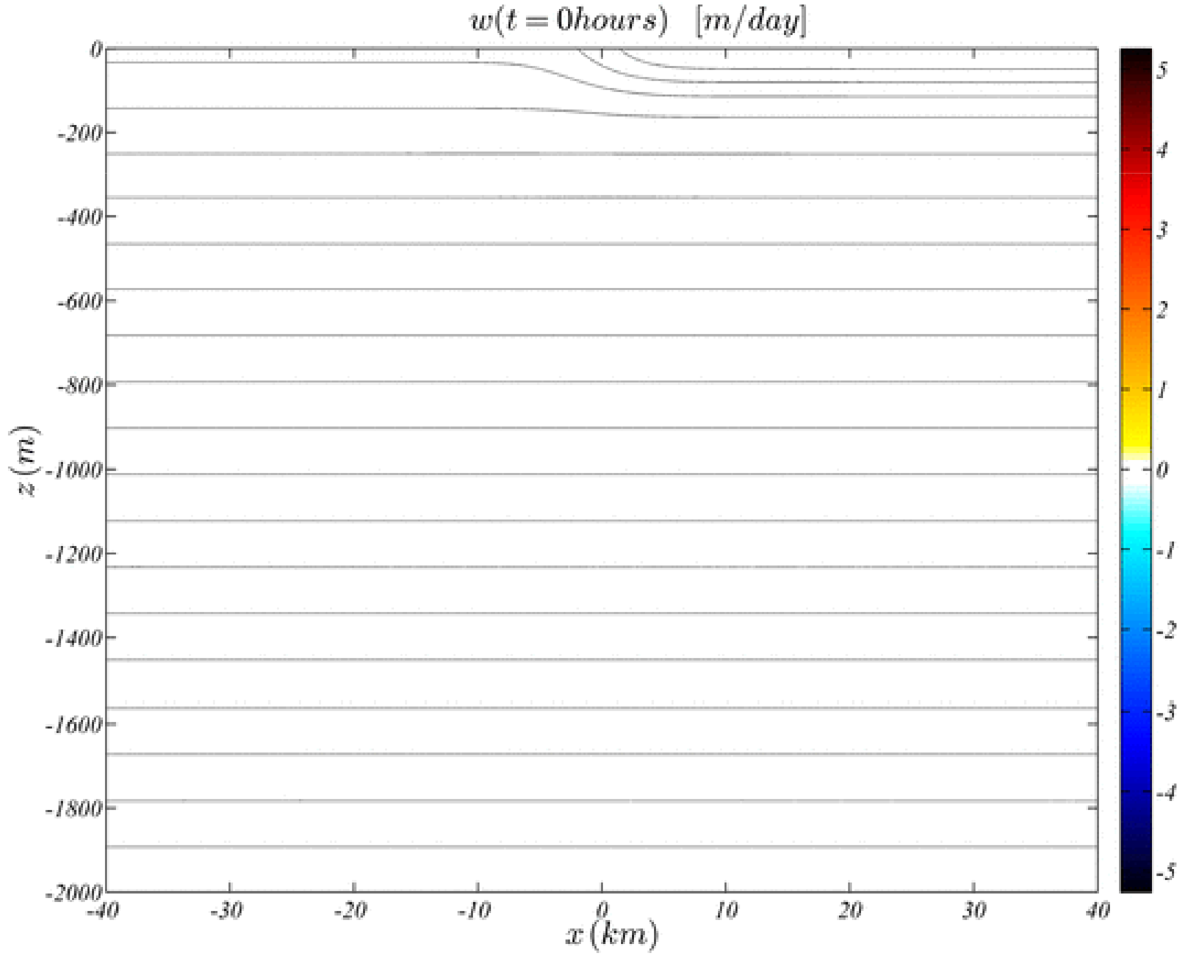
$$b(x) = \frac{\Delta b}{2} \operatorname{erf}\left(\frac{x}{L_s}\right)$$

$$L_s = \sqrt{\frac{2\kappa_h}{\alpha}} = 707\, m$$



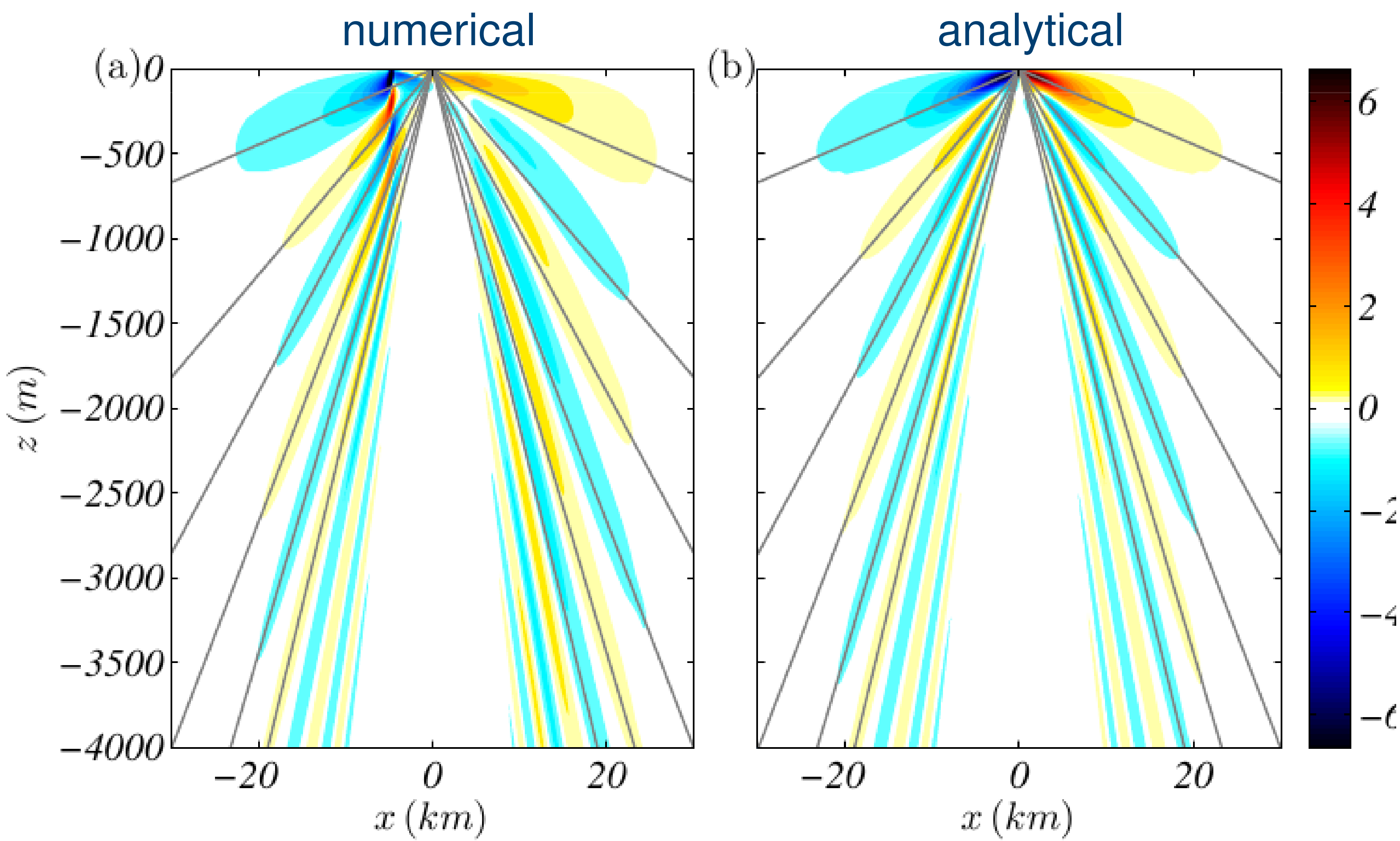
Numerical simulation: MITgcm

- secondary circulation develops
- followed by waves as the strain increases and frontal scale collapses

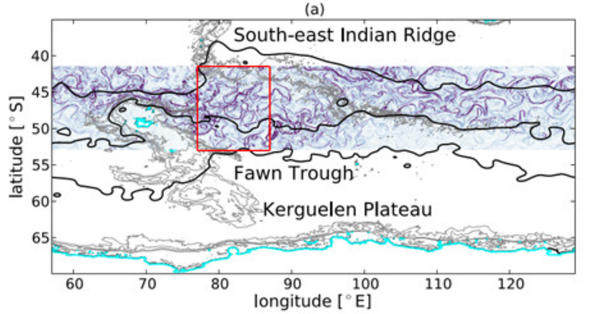


Numerical/analytical comparison: final state

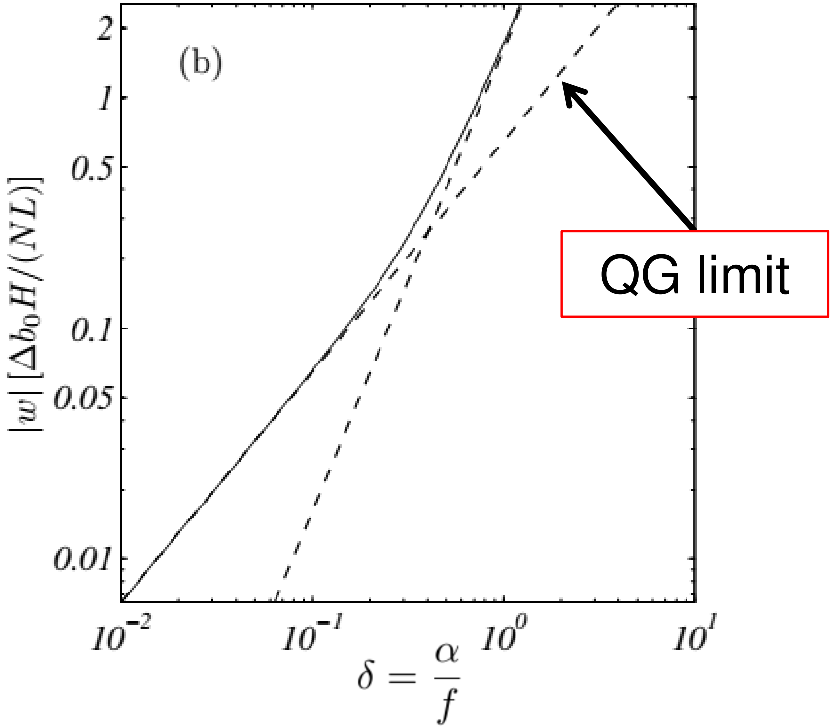
- t=56 hours vertical velocity [m/day]



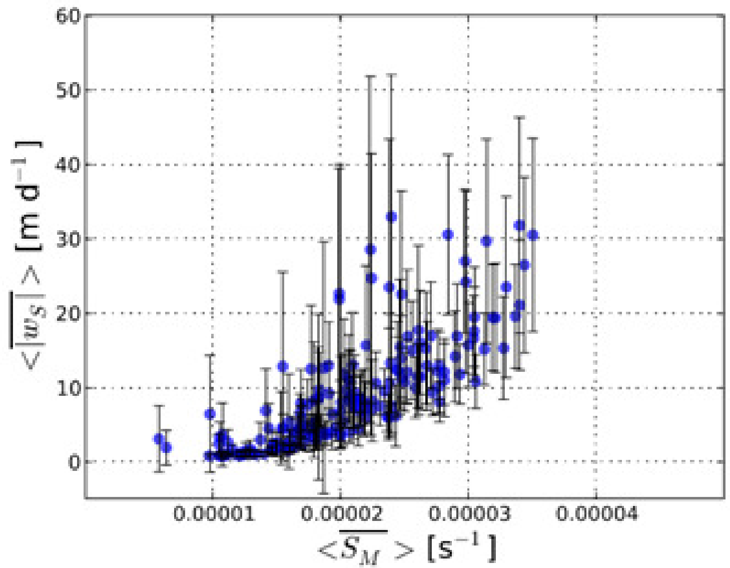
Strength of the secondary circulation (vertical jets near the surface)



<u>Our analytic model:</u>
Vertical velocity magnitude at front as a function of large-scale strain



<u>Ocean sector model: Rosso et al. 2015</u>
Vertical velocity magnitude at submesoscale front as a function of mesoscale strain



** Non-linear effects are also important – see John Taylor's talk on Friday

Conclusions

A new mechanism for wave generation:

- Spontaneous wave generation at strained internal fronts
- Analogous to wave generation due to flow over an obstacle (lee waves)
- Here the obstacle is “internal” = the front

Significant generation for fronts with

- width order of the Rossby radius, approx. $L \leq 5L_R$
- large strains, approx. $\delta \geq 0.2$ (exponential cut-off)

Further information:

- *Spontaneous wave generation at strongly strained density fronts*, Shakespeare and Taylor (submitted), JPO